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Prime Cordial Labeling of the graph P (a, b)

K. Gayathri[1] and C. Sekar[2]

[1]Research Scholar, Periyar Maniammai Institute of Science and Technology, Vallam, Thanjavur, India.

[2]Professor of Mathematics, P.G. Extension Centre, Manonmaniam Sundaranar University, Nakkalmadam, Kanyakumari Dt, India.

*Corresponding Author E-mail: vcdulaganathan@gmail.com

Abstract

A bijection from the vertex set V of a graph G to {1,2,……|V|} is called a prime cordial labeling of G if each edge uv is assigned the label 1 if gcd (f(u), f(v))=1 and 0 if gcd (f(u), f(v))>1, where the number of edges labeled with 0 and the number of edges labeled with 1 differ by at most 1.

Keywords: Prime Cordial Labeling

Subject Classification code: 05C78

Introduction

Graph Labeling have enormous applications within mathematics as well as to several areas of computer science and communication networks. A useful survey to know about the numerous graph labeling methods is given by J.A. Gallian (Gallian, 2012). By combining the relatively prime concept in number theory and cordial labeling concept (Cahit, 1987) in graph labeling, Sundaram *et al*. (Sundaram *et al*., 2005) introduced the concept called prime cordial labeling.

A bijection f from vertex set V (G) to {1, 2 ,……|V(G)|} of a graph G is called a prime cordial labeling of G if for each edge e= uv ∈ E,

$$f^*(e=uv)=1; \text{ if } \gcd(f(u),f(v))=1$$

$$=0; \text{ if } \gcd(f(u),f(v))>1$$

then $|e_f(0) - e_f(1)| \leq 1$, where $e_f(0)$ is the number of edges labeled with 0 and $e_f(1)$ is the number of edges labeled with 1.

In (Baskar Babujee and Babitha, 2013), (Baskar Babujee and Shobana, 2009), (Seoud and Salim, 2010) the following graphs are proved to have prime cordial labeling: c_n if and only if $n \geq 6$; P_n if and only if $n \neq 3$ or 5;$k_{1,n}$(n odd); the graph obtained by subdividing each edge of $k_{1,n}$ if and only if $n \geq 3$.

Preliminaries

The following theorems are proved by G.V.Ghodasara, J.P.Jena (Ghodasara, 2014).

Theorems 2.1

The graph G obtained by joining two copies of the Petersen graph by a path of arbitrary length is prime cordial.

Theorem 2.2

The graph G obtained by joining two copies of fan graph F_n by a path of arbitrary length is prime cordial.

Theorem 2.3

The graph G obtained by joining two copies of flower graph Fl_n by a path of arbitrary length is prime cordial.

Theorem 2.4

The graph G obtained by joining two copies of cycle graph C_n with a triangle by a path of arbitrary length is prime cordial.

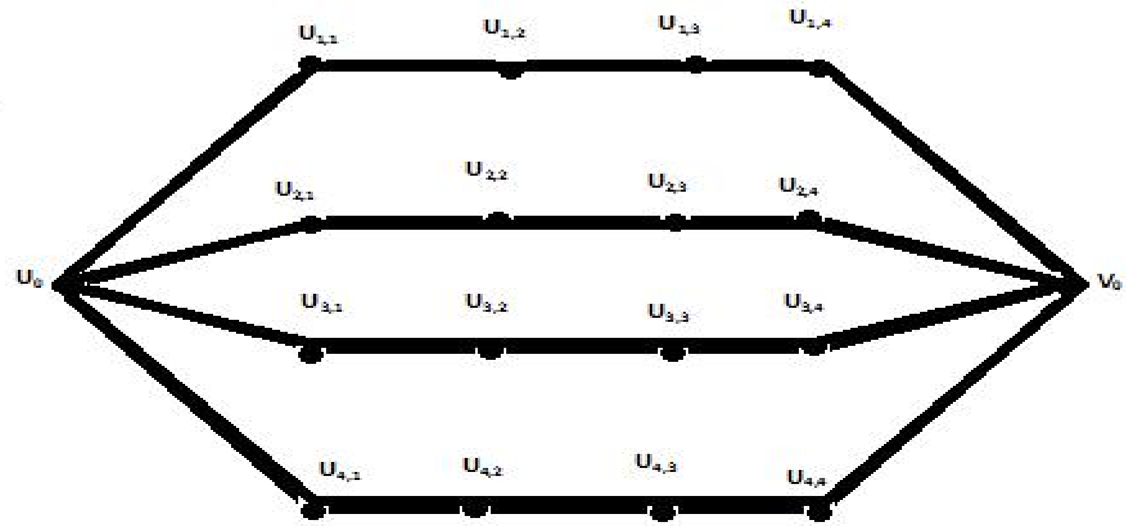
Prime Cordial Labeling of graph P (a, 4)

Definition 3.1

Let u and v be two fixed vertices. We connect u and v by means of “b” internally disjoint paths of length ”a”. The resulting graph is denoted by P (a, b).

Example 3.2

The vertices of P (5,4) are labeled as follows,



In this paper we prove that P(a,b) has Prime Cordial Labeling when b=4.

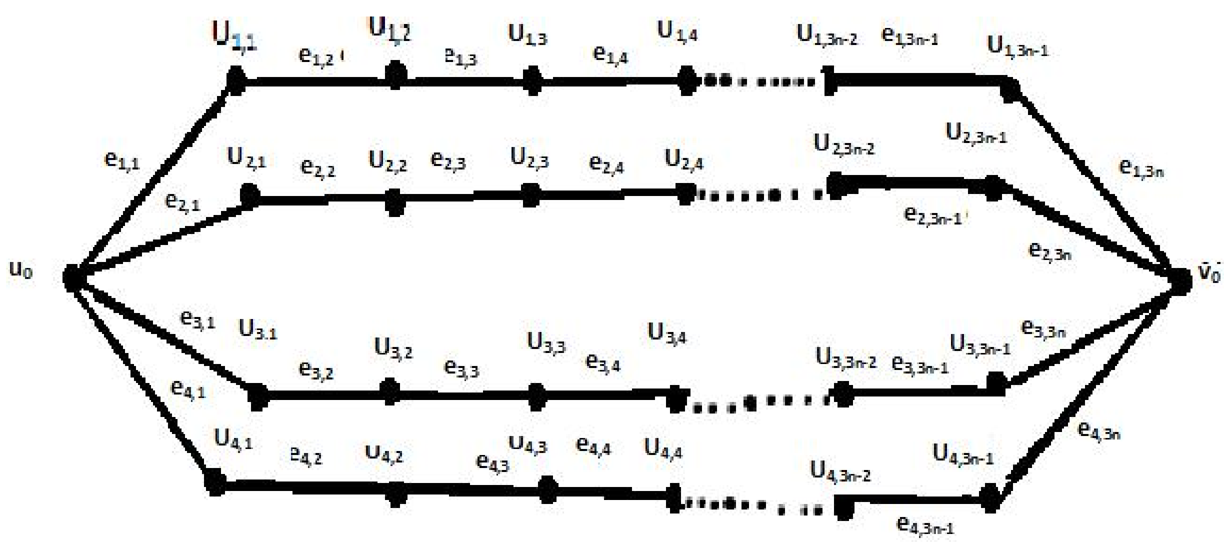
Theorem 3.3

The graph P(a,4) has prime cordial labeling.

Proof:

Case 1: a=3n,n=1,2,3……

We name the vertices of the graph P(3n,4) as follows:



This graph has 4(3n-1) +2=12n-2 vertices and 12n edges. Let $e_{1,1}$, $e_{2,1}$, $e_{3,1}$ and $e_{4,1}$ be the edges joining the vertex u_0 with $u_{1,1}$, $u_{2,1}$, $u_{3,1}$ and $u_{4,1}$ respectively. Similarly let $e_{1,3n}$, $e_{2,3n}$, $e_{3,3n}$ and $e_{4,3n}$ be the edges joining the vertex v_0 with $u_{1,3n-1}$, $u_{2,3n-1}$, $u_{3,3n-1}$, and $u_{4,3n-1}$ respectively.

Let $e_{i,j}$ be the edge joining $u_{i,j-1}$ and $u_{i,j}$ for i=1,2,3,4 and j=2,3,…3n-1

Define f: V(G)→{1,2,…..12n-2} as follows:

$f(u_0)=12n-2$

$f(v_0)=12n-3$

$f(u_{1,j})=2j-1$, for j=1,2,…..3n-1

$f(u_{2,j})=2j$, for j=1,2,….3n-1

$f(u_{3,j})=6n-2+2j-1$, for j=1,2,…3n-1

$f(u_{4,j})=6n-2+2j$, for j=1,2,…3n-1

Clearly, f is one-one.

It is clear that,

$f^*(e_{1,1})=1$

$f^*(e_{2,1})=0$

$f^*(e_{3,1})=0$

$f^*(e_{4,1})=0$

$f^*(e_{1,3n})=0$

$f^*(e_{2,3n})=1$

$f^*(e_{3,3n})=1$

$f^*(e_{4,3n})=1$

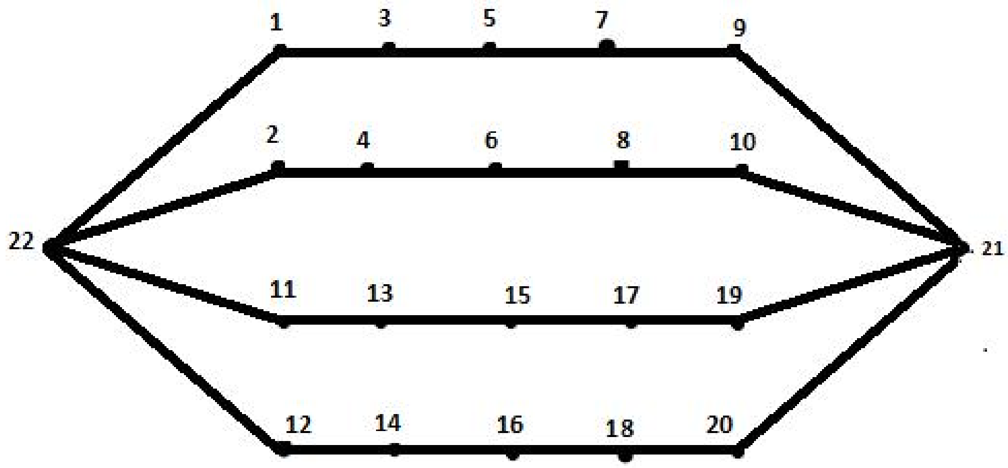
$f^*(e_{i,j})=1$ for i=1,3 and j=2,3,…3n-1

$f^*(e_{i,j})=0$ for i=2,4 and j=2,3,….3n-1

$\therefore |e_f(0)-e_f(1)|=0\leq 1$

This shows that the graph P(3n,4) has a prime cordial labeling.

Example 3.4 Prime Cordial Labeling of P(6,4)

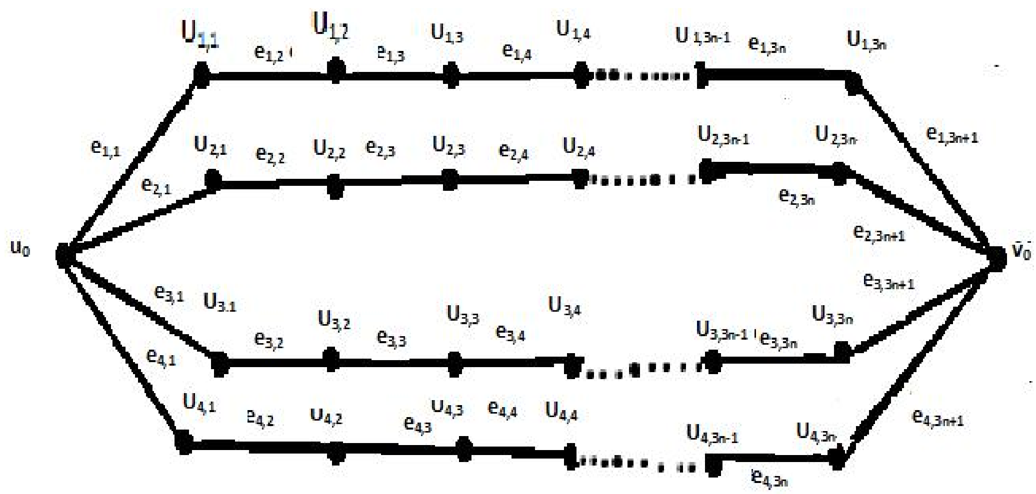


$e_f(0)=12$, $e_f(1)=12$

Case 2: a=3n+1, n=1,2,3,…..

Proof:

We name the vertices of the graph P(3n+1,4) as follows,



This graph has 12n+2 vertices and 12n+4 edges. Let $e_{1,1}$, $e_{2,1}$, $e_{3,1}$ and $e_{4,1}$ be the edges joining the vertex u_0 with $u_{1,1}$, $u_{2,1}$,$u_{3,1}$ and $u_{4,1}$ respectively. Similarly let $e_{1,3n+1}$, $e_{2,3n+1}$, $e_{3,3n+1}$ and $e_{4,3n+1}$ be the edges joining the vertex v_0 with $u_{1,3n}$,$u_{2,3n}$,$u_{3,3n}$, and $u_{4,3n}$ respectively.

Let $e_{i,j}$ be the edge joining $u_{i,j-1}$ and $u_{i,j}$ for i=1,2,3,4 and j=2,3,…3n

Define f: V (G)→{1,2,…..12n+2} as follows:

$f(u_0)=6n+2$

$f(v_0)=12n-3$

$f(u_{1,j})=2j-1$, for j=1,2,…..3n

$f(u_{2,j})=2j$, for j=1,2,….3n

$f(u_{3,j})=6n-1+2j$,for j=1,2,…3n-2

$f(u_{3,j})=6n+1+2j$, for j=3n-1,3n

$f(u_{4,1})=12n+2$

$f(u_{4,j})=6n+2j$, for j=2,3,…3n

Clearly, *f* is one-one.

It is clear that,

$f^*(e_{1,1})=1$

$f^*(e_{2,1})=0$

$f^*(e_{3,1})=1$

$f^*(e_{4,1})=0$

$f^*(e_{1,3n+1})=1$

$f^*(e_{2,3n+1})=0$

$f^*(e_{3,3n+1})=1$

$f^*(e_{4,3n+1})=0$

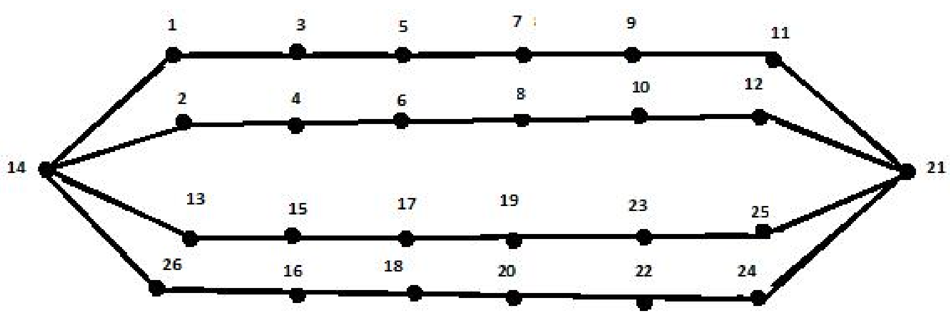
$f^*(e_{i,j})=1$ for i=1,3 and j=2,3,…3n

$f^*(e_{i,j})=0$ for i=2,4 and j=2,3,….3n

$\therefore |e_f(0)- e_f(1)|=0\leq 1$

This shows that the graph P (3n+1,4) has a prime cordial labeling.

Example 3.5 Prime Cordial Labeling of P(7,4)

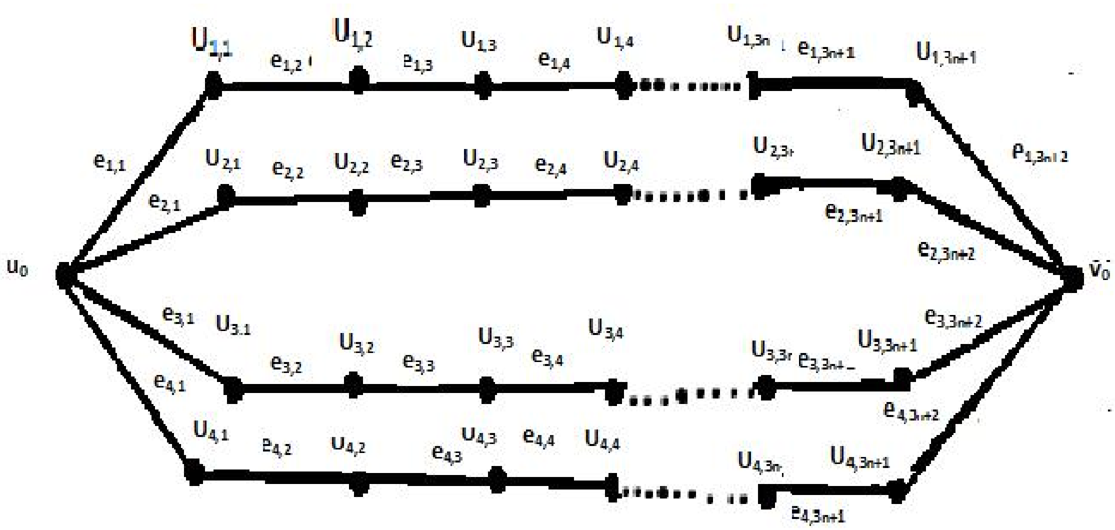


$e_f(0)=14$, $e_f(1)=14$

Case 3: a=3n+2,n=1,2,3,…..

Proof:

We name the vertices of the graph P(3n+2,4) as follows,



This graph has 12n+6 vertices and 12n+8 edges. Let $e_{1,1}$, $e_{2,1}$, $e_{3,1}$ and $e_{4,1}$ be the edges joining the vertex u_0 with $u_{1,1}$, $u_{2,1}$, $u_{3,1}$ and $u_{4,1}$ respectively. Similarly let $e_{1,3n+2}$, $e_{2,3n+2}$, $e_{3,3n+2}$ and $e_{4,3n+2}$ be the edges joining the vertex v_0 with $u_{1,3n+1}$, $u_{2,3n+1,}$ $u_{3,3n+1}$, and $u_{4,3n+1}$ respectively.

Let $e_{i,j}$ be the edge joining $u_{i,j-1}$ and $u_{I,j}$ for i=1,2,3,4 and j=2,3,…3n+1

Define f:V(G)→{1,2,…..12n+6} as follows:

$f(u_0)$=6n+3

$f(v_0)$=12n+2

$f(u_{1,j})$=2j-1, for j=1,2,…..3n+1

$f(u_{2,j})$=2j, for j=1,2,….3n+1

$f(u_{3,1})$=12n+3

$f(u_{3,j})$=6n+1+2j for j=2,3,…..,3n

$f(u_{3,3n+1})$=12n+5

$f(u_{4,j})$=6n+2+2j for j=1,2,..…,3n+1

$f(u_{4,j})$=6n+4+2j, for j=3n,3n+1

Clearly, *f* is one-one.

It is clear that,

$f^*(e_{1,1})$=1

$f^*(e_{2,1})$=1

$f^*(e_{3,1})$=0

$f^*(e_{4,1})$=1

$f^*(e_{1,3n+2})$=0

$f^*(e_{2,3n+2})=0$

$f^*(e_{3,3n+2})=1$

$f^*(e_{4,3n+2})=0$

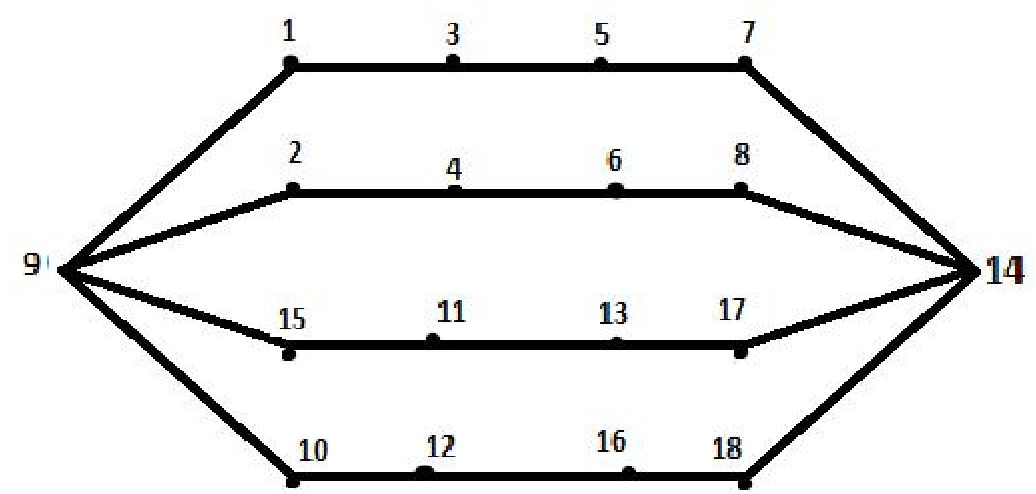
$f^*(e_{i,j})=1$ for i=1,3 and j=2,3,…3n+1

$f^*(e_{i,j})=0$ for i=2,4 and j=2,3,…3n+1

$\therefore |e_f(0)-e_f(1)|=0\leq 1$

This shows that the graph P(3n+2,4) has a prime cordial labeling.

Example 3.6 Prime Cordial Labeling of P(5,4)



$e_f(0)=10$, $e_f(1)=10$

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