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The P Value of likely extreme events

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Abstract

Objective: Sometimes there are circumstances where it is necessary to calculate the P Value of extremely events x_t like $p(x_t) = 1$ while reliable methods are rare.

Methods: A systematic approach to the problem of the P Values of extremely events is provided.

Results: New theorems for calculating P Values of extremely likely events are developed.

Conclusions: It is possible to calculate the P Values even of extreme events.

Keywords: P Value, likely events, cause, effect, causal relationship.

1. Introduction

Assume a binomial experiment while each experiment is called a Bernoulli trial *t* with repeated trials yielding only two possible outcomes: success, S = +1 or failure, F = +0 while the values of $\pi(x_t)$ and $q(x_t) = (1- \pi(x_t))$ may remain unchanged throughout each trial. The probability of success in the population is π and the probability of failure in the population is $q(x_t) = (1- \pi(x_t))$. The binomial distribution was derived by the prominent Suisse mathematician Jacob Bernoulli (1655 - 1705) in his work *Ars Conjectandi* (Bernoulli, 1713) as

$$\mathrm{p}(X = x) \quad = \quad \left(\frac{n!}{x! \times (n-x)!}\right) \pi(x_t)^x \times \left(1 - \pi(x_t)\right)^{n-x} \qquad (1)$$

where x is the number of successes in a sequence of n independent experiments or *Bernoulli* (Uspensky, 1937, p. 45) trials. In particular, what is *the P Value* (Ronald Aylmer Fisher, 1926) under conditions where n=x?

2. Material and methods

2.1. Material

2.1.1. Definitions

Definition 1. (The Binomial distribution)

Let the binomial random variable X denote the number of successes in *n* such Bernoulli trials, where X = 0, 1, 2, 3, ... , n. The expected value of the binomial random variable X is

$$E(X) = E(x_1 + \cdots + x_n) = E(x_1) + \cdots + E(x_n) = \underbrace{(1 + \cdots + 1)}_{n\ times} \times \pi(x_t) = n \times \pi(x_t) \quad (2)$$

The variance of the binomial random variable X is $\sigma(X)^2 = n \times \pi(x_t) \times (1 - \pi(x_t))$.

Proof.

Since $\sigma(x_t)^2 = \pi(x_t) \times (1 - \pi(x_t))$, we get:

$$\sigma(X)^2 = \sigma(x_1 + \cdots + x_n)^2 = \underbrace{\sigma(x_1)^2 + \cdots + \sigma(x_n)^2}_{n\ times} = n \times \sigma(x_t)^2 = n \times \pi(x_t) \times (1 - \pi(x_t)) \quad (3)$$

Q. e. d.

The sample proportion or the relative frequency of an event $p(x_t) = (x / n)$ is the number x of times the event occurred in an experiment or study of *the sample size n* while $x = x_1 + \cdots + x_n$ where all x_t are independently distributed Bernoulli random variables.

$$E\left(\frac{x}{n}\right) = E(p(x_t)) = \frac{E(X)}{n} = \frac{1}{n} \times n \times \pi(x_t) = \pi(x_t) \quad (4)$$

and the sample variance is defined as

$$\sigma\left(\frac{x}{n}\right)^2 = \sigma(p(x_t))^2 = \frac{\sigma(x)^2}{n^2} = \frac{n \times \pi(x_t) \times (1 - \pi(x_t))}{n \times n} = \frac{\pi(x_t) \times (1 - \pi(x_t))}{n} \quad (5)$$

Let p(X = x) denote the probability mass function of observing exactly *x* successes in *n* trials, with the probability of success on a single trial denoted by $\pi(x_t)$ and $q(x_t) = 1 - \pi(x_t)$ is defined as

$$p(X = x) = \left(\frac{n!}{x! \times (n - x)!}\right) \pi(x_t)^x \times (1 - \pi(x_t))^{n-x} \quad (6)$$

In a slightly different way, the definition of binomial distribution does not rule out another distribution derived from the same. Facts taken together suggest the following form derived from the Bernoulli distribution as

$$p(X = x) = \left(\frac{n!}{(x + 1)! \times (n - (x + 1))!}\right) (\pi(x_t))^x \times (1 - \pi(x_t))^{n-x} \quad (7)$$

A binomial distribution with parameters $\pi(x_t)$ and n = 1 is called the *Bernoulli distribution* too while x can take the values either +0 or +1. It is

$$p(X = x) \quad = \quad \left(1 - q(x_t)\right)^{x} \times \left(1 - \pi(x_t)\right)^{1-x} \tag{8}$$

Even if it is some evidence that **0!=1!** (Barukčić, 2019b, p. 195) is not correct, today's rules demand that **0!=1**. Thus far, under conditions where **X = 0** the binomial distribution changes to

$$\begin{aligned} p(X = 0) \quad &= \quad \left(\frac{n!}{x! \times (n-x)!}\right)\left(1 - q(x_t)\right)^{x} \times \left(1 - \pi(x_t)\right)^{n-x} \\ &= \left(\frac{n!}{0! \times (n-0)!}\right)\left(1 - q(x_t)\right)^{0} \times \left(1 - \pi(x_t)\right)^{n-0} \\ &= \left(1 - \pi(x_t)\right)^{n} \end{aligned} \tag{9}$$

3. Results

THEOREM 1. THE PROBABILITY OR PROPORTION OF A LIKELY EVENT

CLAIM.

Under some circumstances, the *proportion or the probability* $\pi(x_t)$ is given approximately by the formula

$$\pi(x_t) \quad = \quad e^{-\left(\frac{E(X_t)}{n}\right)} \tag{10}$$

PROOF.

In general, it is

$$\pi(x_t) \quad = \quad \pi(x_t) \tag{11}$$

where $\pi(x_t)$ denotes the probability of an event x at the *Bernoulli* (Uspensky, 1937, p. 45) trial (period of time) t. Equally likely events are those events which have an equal probability or have the same chance of occurring. Thus far, under conditions where *the probability of an event is constant* from *Bernoulli trial* to Bernoulli trial, we obtain

$$\pi(x_t) \quad = \quad \pi(x_t)^{\frac{\boldsymbol{n}}{\boldsymbol{1}} \times \frac{\boldsymbol{1}}{\boldsymbol{n}}} \tag{12}$$

In particular, it is $\pi(x_t) = 1 - q(x_t)$. Substituting, we obtain

$$\pi(x_t) \quad = \quad \left(\boldsymbol{1} - \boldsymbol{q}(\boldsymbol{x_t})\right)^{n \times \frac{1}{n}} \tag{13}$$

or

$$\pi(x_t) \quad = \quad \left(1 - \frac{\boldsymbol{n} \times q(x_t)}{\boldsymbol{n}}\right)^{n \times \frac{1}{n}} \tag{14}$$

We define E($\underline{X}$) = n×q(x_t). The equation before simplifies as

$$\pi(x_t) \quad = \quad \left(1 - \frac{\boldsymbol{E}(\underline{\boldsymbol{X}}\ )}{n}\right)^{n \times \frac{1}{n}} \tag{15}$$

Increasing the number of randomly generated variables (sample size *n* grows) enable us to take the limit. In point of fact, taking the limit of the term

$$\left(1 - \left(\frac{E(\underline{X}\ )}{n}\right)\right)^{n} \tag{16}$$

as the number of (Bernoulli) trials or the sample size *n* goes to positive infinity ($n \to +\infty$),

$$\lim_{n \to +\infty} \left(\left(1 - \left(\frac{E(\underline{X}\ )}{n}\right)\right)^{n}\right) \tag{17}$$

we obtain according to the known elementary calculus (DeGroot, Schervish, Fang, Lu, & Li, 2005)

$$\lim_{n \to +\infty} \left(\left(1 - \left(\frac{E(\underline{X}\ )}{n}\right)\right)^{n}\right) \quad = \quad e^{-\left(\frac{E(\underline{X}\ )}{1}\right)} \tag{18}$$

Thus far, as the sample size increases or as the number of trials *n* goes to positive infinity ($n \to +\infty$) the equation above simplifies as

$$\pi(x_t) \quad = \quad e^{-\left(\frac{E(\underline{X}\ )}{n}\right)} \tag{19}$$

QUOD ERAT DEMONSTRANDUM.

Under some circumstances, the P Value of ($X \leq n-1$) can be calculated as

$$p(X \leq \mathrm{n} - 1) \quad \equiv \quad 1 \quad - \quad e^{-\left(\frac{E(\underline{X}\ )}{n}\right)} \tag{20}$$

THEOREM 2. THE PROBABILITY OR PROPORTION OF X=N EVENT

CLAIM.

The *proportion of X=n events* is given by the function:

$$p\ \ (X = n) \ = \ 1^{n} \ = \ +1 \tag{21}$$

PROOF.

The binomial distribution is defined as

$$\mathrm{p}(X = x) \ = \ \left(\frac{n!}{x! \times (n-x)!}\right)\left(\pi(x_t)\right)^{x} \times \left(1 - \pi(x_t)\right)^{n-x} \tag{22}$$

Under circumstances where X = n, we obtain

$$\mathrm{p}(X = \mathrm{n}) \ = \ \left(\frac{n!}{\mathrm{n}! \times (n-n)!}\right)\left(\pi(x_t)\right)^{n} \times \left(1 - \pi(x_t)\right)^{n-\mathrm{n}} \tag{23}$$

or

$$\mathrm{p}(X = \mathrm{n}) \ = \ \left(\pi(x_t)\right)^{n} \tag{24}$$

or in other words

$$\mathrm{p}(X = \mathrm{n}) \ = \ \underbrace{\pi(x_t) \times \ldots \times \pi(x_t)}_{n - times} \ = \ \pi(x_t)^{n} \tag{25}$$

However, the probability of an event with $\pi(x_t)$ =1 (the conditio per quam relationship, the exclusion relationship, the necessary and sufficient condition relationship, *the conditio sine qua non relationship* et cetera) is

$$\mathrm{p}(X = \mathrm{n}) \ = \ \underbrace{\pi(x_t) \times \ldots \times \pi(x_t)}_{n - times} \ = \ \pi(x_t)^{n} \ = \ 1^{n} \ = \ +1 \tag{26}$$

QUOD ERAT DEMONSTRANDUM.

THEOREM 3. THE ONE-SIDED LEFT TAILED P VALUE

The following figure may illustrate the one-sided left tailed test for likely events.

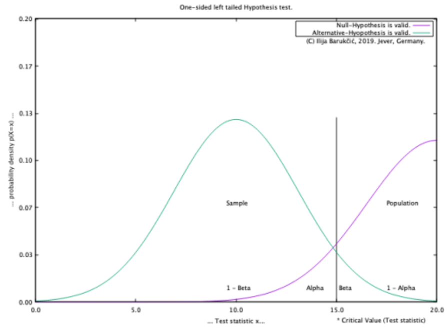

Figure 1. One-sided left tailed test.

A *left tailed* ***P value*** **which is** ***greater than or equal*** **to alpha (α)** or P value ≥ α, provides some evidence (Yamane, 1964) to ***accept the null hypothesis*** (X ≤ n-1) otherwise not. Simplifying, under these circumstances, there is no significant difference between the sample proportion and the population proportion and it is thought that the difference is due to the chance. In other words, the population proportion does not deviate significantly from the sample proportion which itself is less then 1. Thus far, the sample data do not support the claim that there is a significant conditio sine qua non relationship or conditio per quam relationship or exclusion relationship et cetera.

CLAIM.

The one-sided left tailed P value follows as

$$p(X \leq \mathrm{n}-1) \equiv 1 - e^{-\frac{E(\underline{X}\ )}{n}} \tag{27}$$

PROOF.

In general, it is

$$p(X \leq \mathrm{n}-1) + p(X > \mathrm{n}-1) \equiv 1 \tag{28}$$

However, we should consider that the only value where (X > n-1) is the value (X=n) with the consequence that the probabilities are equal or it is p(X > n-1) = p(X=n). The equation before changes to

$$\begin{array}{ccccc} \underbrace{p(X \leq \mathrm{n}-1)} & + & \underbrace{p(X > n-1)} & \equiv & +1 \\ p(X=0)+p(X=1)+\cdots+p(X=\mathrm{n}-1) & + & p(X=n) & \equiv & +1 \end{array} \tag{29}$$

Therefore, the equation before simplifies as

$$p(X \leq \mathrm{n}-1) + p(X = \mathrm{n}) \equiv 1 \tag{30}$$

Considering the equations before, we obtain

$$p(X \leq \mathrm{n}-1) = 1 - p(X = \mathrm{n}) = 1 - p(X > \mathrm{n}-1) \tag{31}$$

As proofed before, it is

$$p(X = \mathrm{n}) = \pi(x_t) = e^{-\left(\frac{E(\underline{X}\ )}{n}\right)} \tag{32}$$

Under these conditions, the left tailed one sided P value for likely events (π = 1) follows as

$$p(X \leq \mathrm{n}-1) \equiv 1 - p(X > \mathrm{n}-1) \equiv 1 - e^{-\frac{E(\underline{X}\ )}{n}} \tag{33}$$

QUOD ERAT DEMONSTRANDUM.

Remark.

For likely but equally extreme value $\pi(x_t) = 1$ it is E(x) = n, the following probability density function p(X=x) could be of use too.

$$p(X = x) = \frac{2}{\sqrt[2]{2 \times \pi \times \sigma^2}} \times e^{-\left(\frac{1}{2} \times \left(\frac{(x - E(x))}{\sigma}\right)^2\right)} \tag{34}$$

In other words, only one half of the Gauss distribution is considered. The following figure may illustrate this distribution.

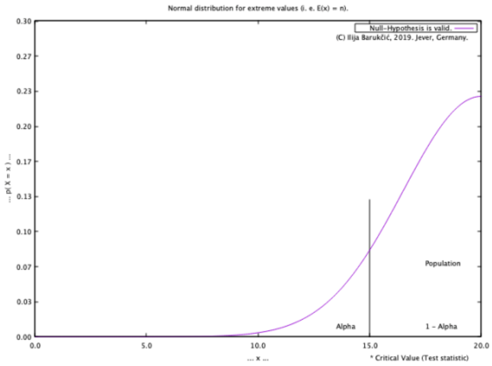

Figure 2. "Normal" distribution for extreme values.

THEOREM 4. EXPONENTIAL DISTRIBUTION AND THE PROBABILITY OF AN EVENT

Let x_t denote a Bernoulli distributed random variable with its own boolean-valued outcome each asking a yes–no question of *either* (success or yes or true or +1) with probability $p(x_t)$ *or* (failure or no or false or +0) with probability $q(x_t) = 1 - p(x_t)$. Let the *probability or the proportion of an event within the population* denoted as $\pi(x_t)$. Let a single success/failure experiment call a *Bernoulli trial t* or Bernoulli experiment and a sequence of outcomes is a Bernoulli process and at the end the sample size *n*.

CLAIM.

The *probability of an event or the proportion of an event within the population* denoted as $\pi(x_t)$ for **E(<u>X</u>) < E(X)** is determined by the number of missed successes E(<u>X</u>) = n - E(X) = 0, 1, ..., *n* in *n* trials and can be calculated by the formula

$$\pi(x_t) = 1 - \left(\frac{E(\underline{X}\ )}{E(X\ )}\right) \times e^{-\left(\frac{E(\underline{X}\ )}{n}\right)} \tag{35}$$

PROOF.

In general, we expect the probability or the population proportion of a binomial distributed random variable in the population, $\pi(x_t)$, is constant from trail to trial and independent of the number of trials performed or it is

$$\pi(x_t) = \pi(x_t) \tag{36}$$

Thus far it is equally valid that

$$\pi(x_t) = \pi(x_t) \ -1 \ +1 \tag{37}$$

or at the end

$$\pi(x_t) = 1 - (1 - \pi(x_t)) \tag{38}$$

Rearranging, we obtain

$$\pi(x_t) = 1 - \frac{\boldsymbol{n} \times \boldsymbol{\pi}(\boldsymbol{x_t}) \times (1 - \pi(x_t))}{\boldsymbol{n} \times \boldsymbol{\pi}(\boldsymbol{x_t})} \tag{39}$$

or

$$\pi(x_t) = 1 - \frac{n \times (1 - \pi(x_t))}{n \times \pi(x_t)} \times \pi(x_t) \tag{40}$$

With respect to the population, it is $E(X) = n \times \pi(x_t)$ and $E(\underline{X}) = n \times (1 - \pi(x_t)) = n \times q(x_t)$. The equation before changes to

$$\pi(x_t) = +1 - \frac{E(\underline{X}\ )}{E(X\ )} \times \pi(x_t) \tag{41}$$

Mathematically, this equation is equivalent with

$$\pi(x_t) = +1 - \frac{E(\underline{X}\ )}{E(X\ )} \times \pi(x_t)^{\frac{\boldsymbol{n}}{\boldsymbol{1}} \times \frac{\boldsymbol{1}}{\boldsymbol{n}}} \tag{42}$$

In general, it is $\pi(x_t) = 1 - q(x_t)$. Substituting, we obtain

$$\pi(x_t) = +1 - \left(\frac{E(\underline{X}\ )}{E(X\ )}\right) \times \left((\boldsymbol{1} - \boldsymbol{q}(\boldsymbol{x_t}))^{\frac{n}{1} \times \frac{1}{n}}\right) \tag{43}$$

or

$$\pi(x_t) = +1 - \left(\frac{E(\underline{X})}{E(X)}\right) \times \left(\left(1 - \frac{\boldsymbol{n} \times q(x_t)}{\boldsymbol{n}}\right)^{\frac{n}{1} \times \frac{1}{n}}\right) \qquad (44)$$

or

$$\pi(x_t) = +1 - \left(\frac{E(\underline{X})}{E(X)}\right) \times \left(\left(1 - \frac{E(\underline{X})}{n}\right)^{\frac{n}{1} \times \frac{1}{n}}\right) \qquad (45)$$

Taking the limit as the sample size n or the number of (Bernoulli) trials goes to positive infinity ($n \to +\infty$), we obtain

$$\left(1 - \left(\frac{E(\underline{X})}{n}\right)\right)^{n} = \lim_{n \to +\infty}\left(\left(1 - \left(\frac{E(\underline{X})}{n}\right)\right)^{n}\right) \qquad (46)$$

According to the known elementary calculus (DeGroot et al., 2005) it is

$$\lim_{n \to +\infty}\left(\left(1 - \left(\frac{E(\underline{X})}{n}\right)\right)^{n}\right) = e^{-E(\underline{X})} \qquad (47)$$

Thus far, as the sample size increases or as the number of trials n goes to positive infinity ($n \to +\infty$) the equation above simplifies for **E(X̲) < E(X)** as

$$\pi(x_t) = +1 - \left(\left(\frac{E(\underline{X})}{E(X)}\right) \times \left(e^{-\frac{E(\underline{X})}{n}}\right)\right) \qquad (48)$$

QUOD ERAT DEMONSTRANDUM.

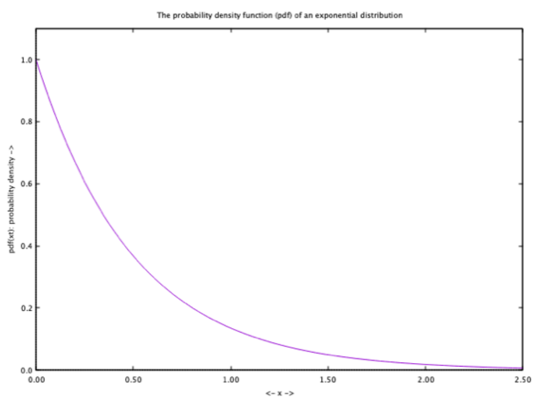

Figure 3. Example exponential distribution

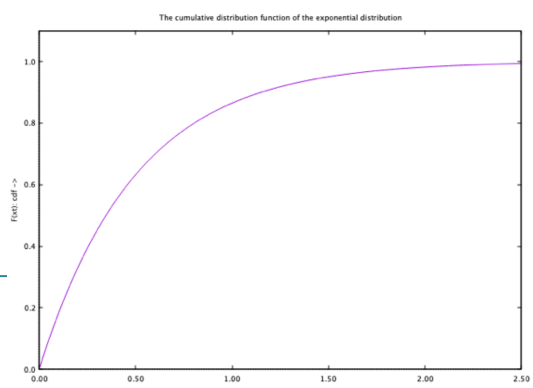

Figure 4. Cumulative distribution function

THEOREM 5. LEFT TAILED ONE-SIDED P VALUE ACCORDING TO EXPONENTIAL DISTRIBUTION

Increasing sample size, it is possible to detect small effects, even though they are not existent in population. The chi square distribution finds its own limits if the sample size is to large or too small. *Fisher's exact test* (Agresti, 1992; R. A. Fisher, 1922; Ronald A. Fisher, 1925) is valid for all sample sizes although in practice it is used when sample sizes are small. The Chi-square (Pearson, 1900) itself is very *sensitive* (Bergh, 2015) to sample size and an extremely large sample is one of the *limitations* (McHugh, 2013) of a Chi-square test. Alternatives to the Chi-Square Test for extremely large samples like *the G-square test* (Sokal & Rohlf, 1995) have been developed to handle large samples in test of fit analysis. However, Chi-square is still reliable with sample size between roughly 100 to 2500 subjects. An exact binomial test can be used when an experiment has two possible outcomes (i.e. success/failure) instead of the chi-square distribution to compare the observed distribution to the expected distribution. The null hypothesis for the binomial test is that the results observed (p(sample proportion)) do not differ significantly from what is expected to be in the population (π = 1). The (*left* tailed) one-sided null and alternative hypotheses may be as follows:

$$\begin{aligned} H_0 &: \quad \pi(population) \quad \leq \quad p(sample\ proportion) \quad (i.e.\ SINE\ relationship : \text{NO}) \\ H_A &: \quad \pi(population) \quad > \quad p(sample\ proportion) \quad (i.e.\ SINE\ relationship : \text{YES}) \end{aligned} \tag{49}$$

How likely is it that an observed difference from what is expected to be is only due to chance? Folowing the Fisherian and Neyman-Pearsonian schools of hypothesis testing the calculation of the P value (Arbuthnott, 1710; Ronald A. Fisher, 1925; Heyde & Seneta, 2001; LaPlace, Pierre Simon de, 1812; Pearson, 1900) can answer questions like these. Since Fisher's statement years ago, it has become ritualistic by medical researchers worldwide to use 0.05 as cut-off for a P value. However, after the advent of computers and statistical software, calculating exact P values is easy now and so the researcher can report exact P values and leave it to a reader to determine the significance of the same. In point of fact, P value being a probability can take any value between 0 and 1. Thus far, P values close to 1 suggests no difference between what is observed from what is expected to be due to chance whereas P values close to 0 indicate that a difference observed is unlikely to be due to chance. However, like the test of hypothesis, the P value itself is associated with several *fallacies* (Dahiru, 2008) and depends on several factors, among other on *the distribution used*. P values alone can completely misrepresent (Bertolaccini, Viti, & Terzi, 2016; Dixon, 2003) the evidence provided by sample data and an alternative analytical technique is necessary to be developed. In general, the P value is *the probability of an outcome*, when the null hypothesis is true, which is at least as extreme as the observed.

CLAIM.

Under certain circumstances (i.e. for **E($\underline{X}$)** < **E(X)**), the (*left*-tailed one-sided) P value can be calculated as

$$p(X \leq (\mathrm{n}-1)) \quad \equiv \quad \left(\left(\frac{E(\underline{X}\;)}{E(X\;)}\right) \times \left(e^{-\frac{E(\underline{X}\;)}{n}}\right)\right) \tag{50}$$

PROOF.

In general, it is

$$+1 \quad \equiv \quad p\big(X < (\mathrm{n}-1)\big) \quad + \quad p\big(X = (\mathrm{n}-1)\big) \quad + \quad p\big(X > (\mathrm{n}-1)\big) \tag{51}$$

or

$$+1 \quad \equiv \quad p\big(X \leq (\mathrm{n}-1)\big) \quad + \quad p\big(X > (\mathrm{n}-1)\big) \tag{52}$$

or

$$p\Big(\big(X \leq (\mathrm{n}-1)\big)\Big) \quad \equiv \quad 1 - p\Big(\big(X > (\mathrm{n}-1)\big)\Big) \tag{53}$$

Mathematically, the only value where (**X** > **(n-1))** is **X** = **x** = **n.** Under these conditions, the probabilities are equal or it is **p(X>(n-1))= p(X=n).** The equation before changes to

$$p\Big(\big(X \leq (\mathrm{n}-1)\big)\Big) \quad \equiv \quad 1 - p\Big(\big(X = (\mathrm{n})\big)\Big) \tag{54}$$

As proofed before, in this case we obtain for **E($\underline{X}$)** < **E(X)**

$$p(X = \mathrm{n}) \quad = \quad +1 \quad - \quad \left(\left(\frac{E(\underline{X}\;)}{E(X\;)}\right) \times \left(e^{-\frac{E(\underline{X}\;)}{n}}\right)\right) \tag{55}$$

This formula derived before is valid even for random variables where X =n. Substituting this relationship into the equation before, we obtain

$$p\left(\left(X \leq (\mathrm{n}-1)\right)\right) \quad \equiv \quad 1 \quad - \quad \left(1-\left(\left(\frac{E\left(\underline{X}\ \right)}{E\left(X\ \right)}\right)\times\left(e^{-\frac{E\left(\underline{X}\ \right)}{n}}\right)\right)\right) \qquad (56)$$

The P value can be calculated according to the equation before (for *very large samples* too) were π is expected to be 1. A *left tailed* ***P value*** **which is *greater than or equal* to α** (P value ≥ α) provides some evidence (Yamane, 1964) to ***accept the null hypothesis*** otherwise not. Simplifying equation, the left tailed P value follows for **E(<u>X</u>)** < **E(X)** as

$$p\left(\left(X \leq (\mathrm{n}-1)\right)\right) \quad \equiv \quad \left(\left(\frac{E\left(\underline{X}\ \right)}{E\left(X\ \right)}\right)\times\left(e^{-\frac{E\left(\underline{X}\ \right)}{n}}\right)\right) \qquad (57)$$

QUOD ERAT DEMONSTRANDUM.

Example.

To perform a hypothesis test with the distribution before, we must calculate the probability, p, of the observed event and any more extreme event happening. We compare this degree of evidence to the level of significance α. Thus far, if the calculate p is **p ≥ α** then *we do accept the null hypothesis* and reject the alternative hypothesis. Under circumstances where **p < α** we do reject the null hypothesis and *accept the alternative hypothesis*. In other words, the (*left* tailed) one-sided null and alternative hypotheses are

$$\begin{aligned} &H_0: \quad \pi(population) \quad \leq \quad p(sample\ proportion) \quad (i.e.\ SINE\ relationship: \mathrm{NO}) \\ &H_A: \quad \pi(population) \quad > \quad p(sample\ proportion) \quad (i.e.\ SINE\ relationship: \mathrm{YES}) \end{aligned} \qquad (58)$$

while the *probability* can be calculated as

$$p(X \leq \mathrm{n}-1) \quad + \quad p(X > n-1) \quad \equiv \quad 1 \qquad (59)$$

Mathematically, if **X** > **n-1** then it is **X = n**. The equation before changes to

$$p(X \leq \mathrm{n}-1) \quad + \quad p(X = \mathrm{n}) \quad \equiv \quad 1 \qquad (60)$$

and at the end to

$$p(X \leq \mathrm{n}-1) \quad \equiv \quad \sum_{t=+0}^{n-1} p(X = t) \quad \equiv \quad 1 \quad - \quad p(X = \mathrm{n}) \qquad (61)$$

Thus far, we obtain for **E(<u>X</u>)** < **E(X)**

$$p(X \leq \mathrm{n}-1) \quad \equiv \quad \left(\left(\frac{E(\underline{X}\ \ )}{E(X\ \ )}\right) \times \left(e^{-\frac{E(\underline{X}\ \ )}{n}}\right)\right) \tag{62}$$

as the left-tailed P Value for likely events. A *P value* which is less than a chosen significance level α (**P value < α**), suggests that the observed data are potentially inconsistent with the null hypothesis (π(population) ≤ p(sample proportion)) and implicate the conclusion that *the null hypothesis should be rejected and we do accept the alternative hypothesis* which claims that π(population) > p(sample proportion). If the *P value* is greater than or equal to the significance level α (**P value ≥ α**), we fail to reject the null hypothesis. Under these circumstance it is necessary to accept that the null-hypothesis: π(population) ≤ p(sample proportion). The *(left tailed) one-sided* null and alternative hypotheses may again be as follows:

$$\begin{aligned} H_0 &: \quad \pi \quad \leq \quad p(sample\ proportion) \quad (i.e.\ SINE\ relationship{:}\ \mathrm{NO}) \\ H_A &: \quad \pi \quad > \quad p(sample\ proportion) \quad (i.e.\ SINE\ relationship{:}\ \mathrm{YES}) \end{aligned} \tag{63}$$

Under some assumptions, the equation above is just a special case of the exponential distribution. In this context let us define the following. Let

$$\lambda \quad \equiv \quad \left(\frac{E(\underline{X}\ \ )}{E(X\ \ )}\right) \quad and \quad p \equiv \quad \frac{E(X\ \ )}{n} \tag{64}$$

while λ is the parameter of the distribution, often called *the rate parameter*. From this follows that

$$E(\underline{X}\ \ ) \quad = \quad \lambda \times E(X\ \ ) \tag{65}$$

Substituting these relationships into the equation above it is

$$1-\pi(x_t) \quad = \quad \left(\frac{E(\underline{X}\ \ )}{E(X\ \ )}\right) \times \left(e^{-\frac{E(\underline{X}\ \ )}{n}}\right) \quad = \quad \lambda \times \left(e^{-\frac{\lambda \times E(X\ \ )}{n}}\right) \tag{66}$$

The probability density function (pdf) of an exponential distribution for λ > 1 follows as

$$1-\pi(x_t) \quad = \quad \left(\frac{E(\underline{X}\ \ )}{E(X\ \ )}\right) \times \left(e^{-\frac{E(\underline{X}\ \ )}{n}}\right) \quad = \quad \lambda \times \left(e^{-\lambda \times p}\right) \tag{67}$$

THEOREM 6. THE APPROXIMATE CRITICAL VALUE DUE TO NORMAL DISTRIBUTION

Under certain conditions (Barukčić, 2019a), the critical P value i. e. of *the conditio sine qua non relationship* $p_{Critical}$ can be calculated as

$$p_{Critical}(A_t \leftarrow B_t) \equiv e^{-Alpha} \quad (68)$$

where *Alpha* denotes the level of significance i. e. Alpha = 0,05 and *e* denotes Euler's number (Euler, 1736), the base of the natural logarithm. *The rule of three* (Hanley, 1983; Jovanovic & Levy, 1997; Louis, 1981; Rumke, 1975), defined as

$$p_{Critical}(A_t \leftarrow B_t) \equiv 1-\left(\frac{3}{n}\right) \quad (69)$$

where *n* denotes the sample size is another way to calculate approximately the critical value for $n > 50$ (Sachs, 1992). In this context, interval estimation of binomial proportions is one of the most basic problems in statistics. The *Wald interval* is more or less the standard interval for the binomial proportion. The Wald method itself is based on the asymptotically normal approximation to the distribution of the observed sample proportion. However, the standard *Wald interval* has a very poor performance (Agresti & Coull, 1998; DasGupta, Cai, & Brown, 2001), even for a very large sample size. Even an 'exact' confidence interval for the binomial proportion as proposed by Clopper & Pearson 1934 (Clopper & Pearson, 1934) are of a restricted (Blyth & Still, 1983) value due to the very wide interval length. In the following we will demonstrate how the population proportion of *the conditio sine qua non relationship* with some limits can be estimated through the usage of a confidence interval known as a *one-sample proportion* in the Z-interval. Let Z denote the critical value of the standard normal distribution for a level of confidence C = 0,95. Thus far, it is

$$Z\ (A_t \leftarrow B_t) \equiv \frac{1-C}{2} \equiv \frac{1-0{,}95}{2} = 0{,}025 \quad (70)$$

The value for Z of a standard normal bell curve gives an upper tail area of 0.0250 or an area of 1 - 0.0250 = 0.9750. Thus far, for α = 0,05 the (1–(α/2)) quantile of a standard normal distribution is (1– (0,05/2)) or equal to Z = 1,959963985.

CLAIM.

The approximate critical lower value of the necessary condition can be calculated as

$$p_{Lower}(A_t \leftarrow B_t) \equiv 1 - \left(\sqrt[2]{\frac{(4)\times\left(\frac{1}{4}\right)}{n}}\right) \equiv 1-\left(\sqrt[2]{\frac{1}{n}}\right) \quad (71)$$

PROOF.

Hence, based on the formula for the one-sample proportion in the Z-interval, the *upper* critical value of a (1- α) confidence interval of the sample proportion without continuity correction can be calculated according to Wald's method (Wald, 1943) as

$$p_{Upper}(A_t \leftarrow B_t) \equiv p_{Sample}(A_t \leftarrow B_t) + \left(\sqrt[2]{\frac{\left(Z^2_{(\alpha/2)}\right)\times\left(p_{Sample}\right)\times\left(1-p_{Sample}\right)}{n}}\right) \quad (72)$$

where *n* denotes the sample size, $p_{Sample}(A_t \leftarrow B_t)$ denotes the sample proportion of *the conditio sine qua non relationship* determined by the number of successes in *n* Bernoulli trials, Z is the (1–(α/2)) quantile of a standard normal distribution and α is the level of significance. The lower confidence bound can be calculated as

$$p_{Lower}(A_t \leftarrow B_t) \equiv p_{Sample}(A_t \leftarrow B_t) - \left(\sqrt[2]{\frac{\left(Z^2_{(\alpha/2)}\right)\times\left(p_{Sample}(A_t \leftarrow B_t)\right)\times\left(1-p_{Sample}(A_t \leftarrow B_t)\right)}{n}}\right) \quad (73)$$

In general, the maximum value is $(p_{Sample}(A_t \leftarrow B_t)\times(1- p_{Sample}(A_t \leftarrow B_t))) \leq (1/4)$. Let $Z^2 = 4$, and let $\mathbf{p_{Sample}(A_t \leftarrow B_t) = 1}$. A more simple and robust form of the equation before simplifies in contrast to Barukčić (Barukčić, 2018) as

$$p_{Lower}(A_t \leftarrow B_t) \equiv 1 - \left(\sqrt[2]{\frac{(4)\times\left(\frac{1}{4}\right)}{n}}\right) \equiv 1-\left(\sqrt[2]{\frac{1}{n}}\right) \quad (74)$$

and provides an approximate value even for events with $\mathbf{p_{Sample}(A_t \leftarrow B_t) = 1}$.

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THEOREM 7. THE APPROXIMATE CRITICAL VALUE DUE TO THE STANDARD SCORE Z

A distribution of sample means even if same is drawn from a non-normal distribution follows under some circumstances more or less the normal distribution. In this context, the z-score is defined as

$$Z(A_t \leftarrow B_t) \equiv \frac{\left(X(A_t \leftarrow B_t) - \left(E(A_t \leftarrow B_t)\right)\right)}{\sigma(A_t \leftarrow B_t)} \quad (75)$$

where $E(A_t \leftarrow B_t)$ is the mean or the expected value of the population and σ is the standard deviation of the population. Under circumstances where the population mean and the population standard deviation are unknown, the standard score Z may be calculated while using the sample mean and sample standard deviation as estimates of the population values.

CLAIM.

The value of *the proportion of an event within the population* denoted as $\pi(x_t)$ can be calculated *approximately* from the number of missed successes in *n* trials $E(\underline{X}_t) = n - E(X_t) = 0, 1, 2, ..., n$ by the function:

$$\pi(x_t) = +1 - \frac{(\underline{X})^2}{Z(x_t)^2 \times n} \tag{76}$$

PROOF.

In general, it is

$$\pi(x_t) = \pi(x_t) \tag{77}$$

or equally

$$\pi(x_t) = \pi(x_t) + 0 \tag{78}$$

or equally

$$\pi(x_t) = \pi(x_t) + 1 - 1 \tag{79}$$

or equally

$$\pi(x_t) = +1 - (1 - \pi(x_t)) \tag{80}$$

Rearranging equation, it is

$$\pi(x_t) = +1 - \frac{(1 - \pi(x_t)) \times \boldsymbol{n} \times \boldsymbol{\pi(x_t)}}{\boldsymbol{n} \times \boldsymbol{\pi(x_t)}} \tag{81}$$

A binomial random variable regarded; *the variance* is $\sigma(x_t)^2 = n \times \pi(x_t) \times (1 - \pi(x_t))$ while *the expectation value* $E(x_t)$ is defined as $E(x_t) = n \times \pi(x_t)$. The equation before simplifies as

$$\pi(x_t) = +1 - \frac{\boldsymbol{\sigma(x_t)^2}}{\boldsymbol{E(x_t)}} \tag{82}$$

From the definition of the z score, we obtain

$$\sigma(A_t \leftarrow B_t)^2 \equiv \frac{\left(X - \left(E(x_t)\right)\right)^2}{Z(x_t)^2} \qquad (83)$$

The equation before can be rearranged as

$$\pi(x_t) = +1 - \frac{\left(\boldsymbol{X} - \left(\boldsymbol{E}(\boldsymbol{x_t})\right)\right)^2}{\boldsymbol{Z}(\boldsymbol{x_t})^2 \times E(x_t)} \qquad (84)$$

Under conditions where the expectation value is $E(x_t) = n \times \pi(x_t) = n \times 1 = n$, the equation before simplifies as

$$\pi(x_t) = +1 - \frac{(X-n)^2}{Z(x_t)^2 \times n} \qquad (85)$$

Let $\underline{X}$ = N-X denote the number of failures in *n* Bernoulli trials. The value of *the proportion of an event within the population* denoted as $\pi(x_t)$ follows approximately for values $\underline{X}^2 < Z(x_t)^2 \times n$ as

$$\pi(x_t) = +1 - \frac{(\underline{X})^2}{Z(x_t)^2 \times n} \qquad (86)$$

while $Z(x_t)$ is the known Z score.

QUOD ERAT DEMONSTRANDUM.

THEOREM 8. THE APPROXIMATE CRITICAL VALUE

CLAIM.

The *proportion of an event within the population* denoted as $\pi(x_t)$ can be calculated *approximately* from the number of missed successes in *n* trials $E(\underline{X}_t) = n - E(X_t) = 0, 1, 2, ..., n$ by the function:

$$\pi(x_t) = +1 - \frac{\chi^2(x_t)}{E(\underline{X})} \qquad (87)$$

PROOF.

In general, we expect that the observed value of the probability or of the sample proportion $p(x_t)$ of a binomial distributed random variable is not significantly different from the expected value of the probability or of the population proportion of a binomial distributed random variable in the population, $\pi(x_t)$. In other words, it is

$$\pi(x_t) = \pi(x_t) \tag{88}$$

or equally

$$\pi(x_t) = \pi(x_t) + 0 \tag{89}$$

or equally

$$\pi(x_t) = \pi(x_t) + 1 - 1 \tag{90}$$

or equally

$$\pi(x_t) = +1 - \left(1 - \pi(x_t)\right) \tag{91}$$

Rearranging equation, it is

$$\pi(x_t) = +1 - \frac{\left(1 - \pi(x_t)\right) \times \boldsymbol{\left(1 - \pi(x_t)\right) \times n \times n}}{\boldsymbol{\left(1 - \pi(x_t)\right) \times n \times n}} \tag{92}$$

The sample proportion $p(x_t)=(X_t/n)$ is the number of successes X_t over the number of trials n. The expected value $E(p(x_t))$ of the sample proportion is an unbiased estimator of the population proportion $\pi(x_t)$. The variance $\sigma(X_t/n)^2$ of the sample proportion $p(x_t) = X_t/n$ is equal to the variance of X_t divided by n^2, or $\sigma(X_t/n)^2 = (n \times p(x_t) \times (1-p(x_t)))/n^2$ or $\sigma(X_t/n)^2 = (p(x_t) \times (1-p(x_t)))/n$. Thus far, while the size of the sample n increases, the variance of the sample proportion $\sigma(X_t/n)^2$ decreases. The value of the Chi-Square of goodness fit test is defined as

$$\chi^2(x_t) = \frac{\left(n \times \left(1 - \pi(x_t)\right)\right)^2}{n} = \frac{n \times \left(1 - E\left(p(x_t)\right)\right) \times n \times \left(1 - E\left(p(x_t)\right)\right)}{n} \tag{93}$$

Substituting into the equation before, we obtain

$$\pi(x_t) = +1 - \frac{\boldsymbol{\chi^2(x_t)}}{\boldsymbol{\left(1 - \pi(x_t)\right) \times n}} \tag{94}$$

Let $E(\underline{X}) = n \times (1 - E(p(x_t)))$, it is

$$\pi(x_t) = +1 - \frac{\boldsymbol{\chi^2(x_t)}}{\boldsymbol{E(\underline{X}\ )}} \tag{95}$$

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Under these assumptions, the Chi-square follows as

$$\chi^2(x_t) = (1 - \pi(x_t)) \times E(\underline{X}) \qquad (96)$$

The critical value of the conditio sine qua non relationship can be estimated by *the rule of three* (Hanley, 1983; Jovanovic & Levy, 1997; Louis, 1981; Rumke, 1975) too.

4. Discussion

The *P Value* as a kind of threshold probability is commonly used as one among other factors which indicate statistical significance especially of clinical investigations. The P Value can help the researcher to better understand the evidence generated by research studies and whether data do not conform to a null hypothesis and whether a clinical research investigation is trustworthy from a scientific perspective. Importantly, and in the historical context, it was difficult to calculate the P Value of events which are assumed to be sure ($p(x_t) = 1$). This publication has solved this problem.

5. Conclusion

It is possible to calculate the P Values even of extremely likely events like $p(x_t) = 1$.

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The author confirms being the sole contributor of this work and has approved it for publication.

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