



## Comparison of Normal, Gamma and Extreme Value Families of Distributions for Estimation of Peak Flood

N. Vivekanandan

Central Water and Power Research Station, Pune, Maharashtra, India

\*Corresponding author E-mail: [anandaan@rediffmail.com](mailto:anandaan@rediffmail.com)

### Abstract

Estimation of Peak Flood (PF) for a given return period at a desired location on a river is required for planning, design and management of civil and hydraulic structures. This can be achieved by using deterministic models with extreme storm events or through frequency analysis by fitting probability distributions to the series of observed Annual Peak Flood (APF) data. In the latter approach, suitable probability distributions and associated parameter estimation methods are applied. In this paper, the parameters of normal, gamma and extreme value families of probability distributions are determined by method of moments, maximum likelihood method and L-moments, and are used for estimation of PF. The adequacy of fitting probability distributions to the series of observed APF data is quantitatively assessed by applying Goodness-of-Fit (GoF) tests viz., Chi-square and Kolmogorov-Smirnov. In addition to GoF tests, diagnostic test of D-index is used for identifying a suitable probability distribution for estimation of PF. However, the results of GoF and diagnostic tests offered diverging inferences, which leads to adopt qualitative assessment by using fitted curves of the estimated values for the selection of best fit probability distribution for estimation of PF. This paper presents a study on comparison of normal, gamma and extreme value families of probability distributions for estimation of peak flood with illustrative example and the results obtained thereof.

**Keywords:** Chi-square, D-index, Kolmogorov-Smirnov, L-Moments, Method of moments, Maximum likelihood method, Peak flood, Probability distribution

### Introduction

Peak Flood (PF) for a given return period is necessarily to be estimated for planning and design of hydraulic structures such as bridges, barrages, culverts, dams, etc. As the estimated PF is highly stochastic in nature, which can be effectively determined by fitting probability distributions to the

series of observed Annual Peak Flood (APF) data. An APF is the highest instantaneous discharge value at a definite cross-section of a natural stream (or) river for an entire year (Gubareva and Gartsman, 2010). Longer period of observation would offer a longer length of the series which could give better results in Flood Frequency Analysis (FFA).

A number of probability distributions belong to the normal, gamma and extreme value families of distributions will generally be adopted in FFA. The Normal (NOR) family of distributions consists of Normal (NOR), 2-parameter Log-Normal (LN2), 3-parameter Log Normal (LN3) and Generalized Normal while the Gamma (GAM) family of distributions consists of Exponential (EXP), Gamma (GAM), Generalized Gamma, Pearson Type-3 (PR3) and Log Pearson Type-3 (LP3). Likewise, Generalized Extreme Value (GEV), Extreme Value Type-1 (EV1), Extreme Value Type-2 (EV2) and Generalized Pareto (GPA) distributions are the members of Extreme Value family of Distributions (EVD) (Vivekanandan, 2020). Based on the intended applications and the variate under consideration, parameter estimation procedures viz., Method of Moments (MoM) and Maximum Likelihood Method (MLM) are used for determination of parameters of the distributions (Naghavi et al., 1993). Generally, MoM is used in determining the parameters of the probability distributions. Sometimes, it is difficult to assess exact information about the shape of a distribution that is conveyed by its third and higher order moments (Malekinezhad et al., 2011). Also, when the sample size is small, the numerical values of sample moments can be very different from those of the probability distribution from which the sample was drawn. It is also reported that the estimated parameters of distributions fitted by using MoM are often less accurate than those obtained by MLM, method of least squares and probability weighted moments. To address these shortcomings, the application of alternative approach, namely L-Moments (LMO) is used (Hosking, 1990).

Badreldin and Feng (2012) carried out regional FFA for the Luanhe basin using LMO and cluster techniques. Haberlandt and Radtke (2014) carried out FFA using APF data for three mesoscale catchments in northern Germany. Markiewicz et al. (2015) adopted Generalized Exponential (GE) and inverse Gaussian distributions in frequency analysis of annual maximum flows for Polish rivers. They described that the GE occupies as front runner among all distributions commonly used for FFA of Polish data and can be included into the group of the alternative distributions. Kossi et al. (2016) carried out regional FFA for Volta River Basin (VRB) using LMO of five probability distributions. By using LMO diagrams and Goodness-of-Fit (GoF) test (i.e., Z-statistic), they found that the GEV and the GPA distributions are better suited to yield accurate flood quantiles in VRB. Amr et al. (2017) compared the performance of several parameter estimators of GPA distribution through Monte Carlo simulation. Ul Hassan et al. (2019) applied the GEV, PR3, EV1 and LN3 distributions for estimation of flood at five gauging sites of Torne River. Lawrence (2020) carried out the study on uncertainty introduced by FFA in projections for changes in flood magnitudes under a future climate in Norway.

Thus, the studies reported didn't suggest applying a particular distribution for FFA for different region or country. This apart, when different distributions are used for estimation of PF, a common problem is encountered as regards the issue of best model fits for a given set of data. This can be answered by formal statistical procedures involving GoF and diagnostic analysis; and the results are quantifiable and reliable (Zhang, 2002). Qualitative assessment is made from the plots of the observed and estimated PF. For quantitative assessment on PF within in the observed range, GoF tests viz., Chi-square ( $\chi^2$ ) and Kolmogorov-Smirnov (KS) tests are applied. In addition to GoF tests, the diagnostic test viz., D-index is used for identifying the suitable probability distribution for estimation of PF (USWRC, 1982). This paper presents the procedures adopted in selecting a best suitable distribution amongst eight probability distributions studied in FFA using qualitative and quantitative assessments with illustrative example and the results obtained thereof.

## Methodology

The methodology involved in carrying out FFA include (i) select the probability distributions for FFA (viz., NOR, LN2, GAM, EXP, LP3, GEV, EV1 and EV2 distributions); (ii) select parameter estimation methods (say, MoM, MLM and LMO); (iii) select quantitative GoF and diagnostic tests; (iv) carry out quantitative and qualitative assessments and (v) conduct FFA and analyze the results obtained thereof. Table 1 presents the Cumulative Distribution Function (CDF), quantile estimator (q(T)) for a return period 'T' of the probability distributions adopted in FFA (Rao and Hamed, 2000).

## Theoretical Concept of Parameter Estimation Methods

### Method of Moments

MoM is a technique for constructing estimators of the parameters based on matching the sample moments with the corresponding distribution moments (Ghorbani et al., 2010). The  $r^{\text{th}}$  central moment ( $\mu_r$ ) about the mean ( $\bar{q}$ ) of a random variable  $q$  is defined by:

$$\mu_r = E(q - \bar{q})^r = \int (q - \bar{q})^r f(q) dq, \text{ if } q \text{ is continuous variable} \quad \dots (1)$$

where,  $f(q)$  is probability distribution function of a random variable  $q$ . The second moment ( $\mu_2$ ) about  $\bar{q}$  is called as variance. Also, third and fourth moments ( $\mu_3$  and  $\mu_4$ ) about  $\bar{q}$  are called as Coefficient of Skewness ( $C_S$ ) and Coefficient of Kurtosis ( $C_K$ ), which are given as  $C_S = \mu_3 / \mu_2^{3/2}$  and  $C_K = (\mu_4 / \mu_2^2) - 3$  ... (2)

### Maximum Likelihood Method

The MLM is a technique identifying the most likely values of location ( $\xi$ ), scale ( $\alpha$ ) and shape ( $k$ ) parameters of the distribution for a given sample. The method adopts parameter values by maximizing a likelihood function,

$$L(\xi, \alpha, k) = \prod_{i=1}^N f(q_i; \xi, \alpha, k) \quad \dots (3)$$

Generally, L is expressed by the log-likelihood function. Taking the first derivatives of Eq. (3), we get

$$\frac{\partial(L(\xi, \alpha, k))}{\partial(\xi, \alpha, k)} = 0 \quad \dots (4)$$

with respect to the parameters of any probability distribution yields three equations for  $\xi$ ,  $\alpha$  and  $k$  (Vogel and Wilson, 1996). If necessary, the second derivative can be used to determine the sign of the solution.

### *L-Moments*

LMOs are analogous to ordinary moments, which provide measures of location, dispersion, skewness, kurtosis and other aspects of the shape of probability distributions or data samples. But, LMOs are computed from linear combinations of the ordered data values. LMO can be used as the basis of a unified approach to the statistical analysis adopting probability distributions. According to Hosking (1990), LMOs have the following advantages:

- i) It characterizes a wider range of probability distributions than conventional moments.
- ii) It is less sensitive to outliers in the data.
- iii) It approximates their asymptotic normal distribution more closely.
- iv) It is nearly unbiased for all combinations of sample sizes and populations.

LMO thus would be useful in providing accurate quantile estimates of hydrological data in developing countries where small sample size typically exists. LMO is a linear combination of probability weighted moments. Let  $q_1, q_2, \dots, q_N$  be a conceptual random sample of size  $N$  and  $q_1 \leq q_2 \leq \dots \leq q_N$  denote the corresponding order statistics. The  $r+1^{\text{th}}$  LMO ( $\lambda_{r+1}$ ) defined by Hosking and Wallis (1993), is given as below:

$$\lambda_{r+1} = \sum_{k=0}^r \frac{(-1)^{r-k} (r+k)!}{(k!)^2 (r-k)!} b_k \quad \dots (5)$$

wherein  $b_k$  is an unbiased estimator and given by

$$b_k = N^{-1} \sum_{i=k+1}^N \frac{(i-1)(i-2)\dots(i-k)}{(N-1)(N-2)\dots(N-k)} q_i \quad \dots (6)$$

The first three LMOs ( $\lambda_1$ ,  $\lambda_2$  and  $\lambda_3$ ) are expressed by:

$$\lambda_1 = b_0, \lambda_2 = 2b_1 - b_0 \text{ and } \lambda_3 = 6b_2 - 6b_1 + b_0 \quad \dots (7)$$

**Table 1: CDF and Quantile estimator of Normal, Gamma and Extreme Value families of Distributions**

Distribution	CDF	Quantile Estimator (q(T))
NOR	$F(q) = \Phi\left(\frac{q - \mu(q)}{\sigma(q)}\right)$	$q(T) = \mu(q) + \sigma(q) \Phi^{-1}(P)$
LN2	$F(q) = \Phi\left(\frac{\ln(q) - \mu(y)}{\sigma(y)}\right)$	$q(T) = \exp(\mu(y) + \sigma(y) \Phi^{-1}(P))$
GAM	$F(q) = \begin{cases} G\left(k, \frac{q - \xi}{\alpha}\right) & , \alpha > 0 \\ 1 - G\left(k, \frac{q - \xi}{\alpha}\right) & , \alpha < 0 \end{cases}$	No explicit expression of the quantile function is available
EXP	$F(q) = 1 - \exp\left[-\frac{(q - \xi)}{\alpha}\right]$	$q(T) = \xi - \alpha \ln(1 - P)$
LP3	$F(q) = \begin{cases} G\left(k, \frac{\ln(q) - \xi}{\alpha}\right) & , \alpha > 0 \\ 1 - G\left(k, \frac{\ln(q) - \xi}{\alpha}\right) & , \alpha < 0 \end{cases}$	No explicit expression of the quantile function is available
GEV	$F(q) = \exp\left(-\left[1 - \frac{k(q - \xi)}{\alpha}\right]^{1/k}\right), \alpha > 0$	$q(T) = \xi + \frac{\alpha}{k} \left(1 - [-\ln(P)]^k\right)$
EV1	$F(q) = \exp\left(-\exp\left(-\frac{(q - \xi)}{\alpha}\right)\right)$	$q(T) = \xi - \alpha \ln[-\ln(P)]$
EV2	$F(q) = \exp\left(-\left(\frac{q}{\alpha}\right)^{-k}\right)$	$q(T) = \alpha [-\ln(P)]^{-(1/k)}$

Here,  $\xi$  is the location parameter,  $\alpha$  is the scale parameter,  $k$  is the shape parameter,  $\mu(q)$  and  $\sigma(q)$  are the average and standard deviation of the observed data,  $\mu(y)$  and  $\sigma(y)$  are the average and standard deviation of the log transformed series of the observed data (i.e.,  $y = \ln(q)$ ),  $F(q)$  (or  $F$ ) is the CDF of  $q$  (i.e., Annual Peak Flood),  $\Phi^{-1}$  is the inverse of standard normal distribution function,  $\Phi^{-1} = (P^{0.135} - (1 - P)^{0.135}) / 0.1975$  where in  $P$  is the probability of exceedance and  $q(T)$  is the estimated PF for a return period ( $T$ ). A relation between the terms  $F$ ,  $P$  and  $T$  is defined by  $F$  (or  $F(q)$ ) =  $1 - P = 1 - 1/T$ .

### Goodness-of-Fit Tests

GoF tests are essential for checking the adequacy of probability distributions to the APF series in the estimation of PF. Out of a number GoF tests available, the widely accepted GoF tests are  $\chi^2$  and KS (Charles Annis, 2009), which are used in the study. The theoretical descriptions of GoF tests statistic are given as below:

$\chi^2$  test statistic is defined by:

$$\chi^2 = \sum_{j=1}^{NC} \frac{(O_j(q) - E_j(q))^2}{E_j(q)} \quad \dots (8)$$

where,  $O_j(q)$  is the observed frequency value of  $q$  for  $j^{\text{th}}$  class,  $E_j(q)$  is the expected frequency value of  $q$  for  $j^{\text{th}}$  class and  $NC$  is the number of frequency classes. A rejection region of  $\chi^2$  statistic at the desired significance level ( $\eta$ ) is given by  $\chi_C^2 \geq \chi_{1-\eta, NC-m-1}^2$ . Here,  $m$  denotes the number of parameters of distribution and  $\chi_C^2$  is the computed value of statistic by probability distributions.

KS test statistic is defined by:

$$KS = \text{Max}_{i=1}^N (F_e(q_i) - F_D(q_i)) \quad \dots (9)$$

where,  $q_i$  is the observed APF for  $i^{\text{th}}$  sample,  $F_e(q_i) = i/(N+1)$  is the empirical CDF of  $q_i$ ,  $F_D(q_i)$  is the computed CDF of  $q_i$  using probability distribution and  $N$  is the number of sample values.

*Test criteria:* If the computed values of GoF tests statistic given by the distribution are less than that of the theoretical values at the desired level of significance then the distribution is considered to be acceptable for estimation of PF at that level.

### Diagnostic Test

The selection of a suitable probability distribution for estimation of PF is also carried out through D-index. The theoretical expression of D-index is given as below:

$$D\text{-index} = (1/\bar{q}) \sum_{i=1}^6 |q_i - q_i^*| \quad \dots (10)$$

Here,  $\bar{q}$  is the average value of the observed APF whereas  $q_i$  ( $i=1$  to  $6$ ) and  $q_i^*$  are the six highest observed and corresponding estimated PF (USWRC, 1982). The probability distribution with minimum D-index value is identified as better suited for estimation of PF.

### Application

In this paper, a study on estimation of PF at Akhnoor gauging site adopting gamma, normal and extreme value families of probability distributions is carried out. The Akhnoor site is the lower most gauging site in India, which is located in the Chenab river basin that formed after the two streams the Chandra and the Bhaga merge with each other. The catchment area of the river Chenab upto Akhnoor site is 21808 km<sup>2</sup> (CWC, 2010). The APF data series (i.e., 48 years) pertaining to water year (June to May) for the period from 1971-72 to 2018-2019 is extracted from the daily discharge data series and also used in FFA. The descriptive statistics viz., average ( $\mu(q)$ ), standard deviation ( $\sigma(q)$ ), Coefficient of Skewness ( $C_s$ ) and Coefficient Kurtosis ( $C_K$ ) of the observed APF is computed as 5897.1 cumecs, 4713.5 cumecs, 3.521 and 16.635 respectively. For the log-transformed series of observed APF (i.e.,  $y = \ln(q)$ ), the descriptive statistics viz.  $\mu(y)$ ,  $\sigma(y)$ ,  $C_s$  and  $C_K$  are computed as 8.502 cumecs, 0.553 cumecs, 1.038 and 1.223 respectively.

## Results and Discussions

By applying the procedures of FFA, as described above, parameters of eight probability distributions (viz., NOR, LN2, GAM, EXP, LP3, GEV, EV1 and EV2) were determined by MoM, MLM and LMO; and are used for estimation of PF for different return periods. The estimated PF for different return periods by normal, gamma and extreme value families of distributions are presented in Tables 2 to 4 while the plots are shown in Figures 1 to 3.

**Table 2: Estimated Peak Flood (cumecs) using Normal Family of Distributions**

Return period (year)	NOR			LN2		
	MoM	MLM	LMO	MoM	MLM	LMO
2	5897.2	5897.2	5897.2	4606.5	4922.5	4922.5
5	9864.2	9822.6	8912.0	8322.8	7804.6	7842.8
10	11937.8	11874.5	10487.9	11338.6	9930.8	10004.8
20	13650.2	13569.0	11789.4	14637.1	12116.9	12232.9
50	15577.5	15476.2	13254.1	19510.5	15157.9	15339.4
100	16862.4	16747.6	14230.6	23630.7	17598.4	17837.3
200	18038.4	17911.2	15124.3	28159.9	20174.8	20478.2
500	19463.4	19321.4	16207.3	34827.0	23807.6	24208.0
1000	20463.0	20310.5	16967.0	40425.0	26739.5	30414.0

**Table 3: Estimated Peak Flood (cumecs) using Gamma Family of Distributions**

Return period (year)	GAM			EXP			LP3		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
2	4699.8	5240.2	4817.3	4450.8	4927.0	4656.8	4480.6	4772.5	4592.6
5	9081.3	8437.3	9308.4	8769.8	8310.9	8360.6	7467.4	7478.0	7654.1
10	12161.7	10521.1	12465.7	12036.9	10870.6	11162.3	10337.8	10180.1	10596.2
20	15145.0	12467.9	15523.6	15304.1	13430.4	13964.1	13959.7	13680.9	14308.7
50	18996.7	14911.9	19471.6	19623.0	16814.3	17667.8	20281.6	19975.4	20788.6
100	21862.9	16693.8	22409.5	22890.2	19374.0	20469.6	26578.6	26430.6	27243.1
200	24699.5	18433.5	25317.0	26157.4	21933.8	23271.3	34564.4	34833.9	35428.5
500	28414.4	20683.5	29124.8	30476.3	25317.7	26975.0	48473.9	49942.4	49685.7
1000	31203.7	22355.4	31983.8	33743.5	27877.4	29776.8	62264.0	65406.3	63820.6

From Figure 1, it is noted that the estimated PF using LN2 (MoM) gave higher estimates for return periods from 20-year and above. From Figure 2, it is noticed that the fitted curves of the estimated PF using LP3 (using MoM, MLM and LMO) distribution are in the form exponential trend from 50-year return period onwards. From Figure 3, it can be seen that the fitted curves of the estimated PF using GEV and EV2 are in the form of exponential trend, and the plots of EV1 are in the form of linear trend while MoM, MLM and LMO are applied for determination of parameters of GEV, EV1 and EV2.

**Table 4: Estimated Peak Flood (cumecs) using Extreme Value Family of Distributions**

Return period (year)	GEV			EV1			EV2		
	MoM	MLM	LMO	MoM	MLM	LMO	MoM	MLM	LMO
2	4761.5	4444.1	4458.3	5122.5	5129.0	5282.8	4787.5	4479.3	4446.6
5	8293.5	7015.0	7230.4	9289.7	7720.2	8587.6	7207.5	7175.9	7199.5
10	11111.5	9481.0	9937.7	12048.6	9435.8	10775.7	9449.8	9803.4	9905.2
20	14240.9	12650.8	13468.5	14695.1	11081.5	12874.5	12253.5	13223.8	13451.6
50	19019.1	18366.3	19940.6	18120.7	13211.6	15591.3	17152.2	19480.5	19989.9
100	23228.5	24278.3	26741.7	20687.7	14807.9	17627.1	22068.3	26042.0	26898.6
200	28047.6	32054.8	35811.2	23245.4	16398.3	19655.4	28367.5	34776.8	36155.9
500	35518.8	46239.3	52622.8	26619.7	18496.6	22331.5	39508.8	50934.9	53412.4
1000	42145.0	60984.4	70373.8	29169.9	20082.4	24354.0	50749.2	67962.0	71732.9

**Analysis Based on GoF Tests**

By using MoM, MLM and LMO estimators of normal, gamma and extreme value families of distributions, GoF tests statistic values were computed and are presented in Table 5. In the present study, the number of frequency class is considered as 6 while the degrees of freedom is determined as 2 for 3-parameter probability distributions (viz., LP3 and GEV) and 3 for 2-parameter distributions (viz., NOR, LN2, GAM, EXP, EV1 and EV2) while computing the  $\chi^2$  test statistic values.

**Table 5: Theoretical and Computed values of GoF tests statistic by Normal, Gamma and Extreme Value Families of distributions**

Distribution	Theoretical value of GoF tests statistic at 5% level		Computed values of GoF tests statistic					
	$\chi^2$	KS	$\chi^2$			KS		
			MoM	MLM	LMO	MoM	MLM	LMO
NOR	7.815	0.196	35.250	36.750	39.000	0.214	0.221	0.227
LN2	7.815	0.196	9.750	9.650	9.500	0.139	0.135	0.137
GAM	7.815	0.196	22.000	21.000	21.500	0.195	0.205	0.201
EXP	7.815	0.196	23.000	12.250	9.500	0.190	0.151	0.116
LP3	5.991	0.196	3.250	3.500	3.250	0.055	0.075	0.055
GEV	5.991	0.196	13.250	15.250	5.750	0.128	0.095	0.063
EV1	7.815	0.196	20.750	20.250	19.250	0.188	0.171	0.158
EV2	7.815	0.196	11.000	7.525	5.750	0.162	0.112	0.064

From GoF tests results, it is noted that:

- The  $\chi^2$  test results indicated that LP3 (using MoM, MLM and LMO), GEV (LMO) and EV2 (MLM and LMO) distributions are acceptable for FFA for the data considered in the study.
- The KS test results confirmed the applicability of LN2, EXP, LP3, GEV, EV1 and EV2 distributions for FFA while MoM, MLM and LMO are applied for determining the estimators of the parameters of the distributions.
- The KS test results indicated that the GAM (MoM) distribution is acceptable for FFA.



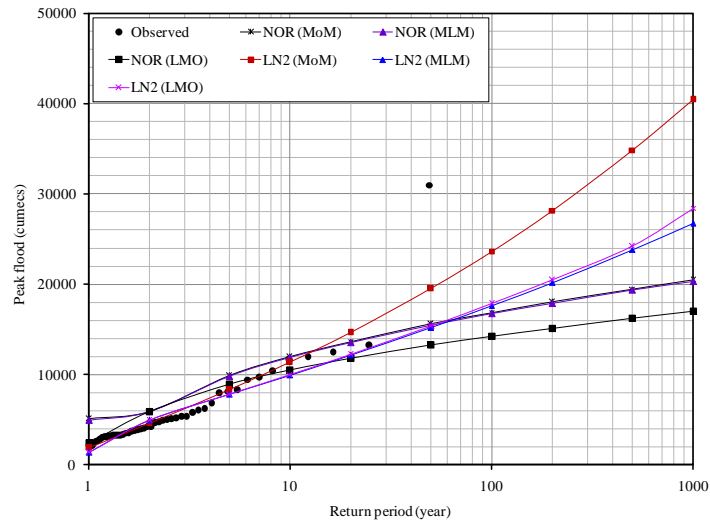


Figure 1: Estimated PF using normal family of distributions with observed APF

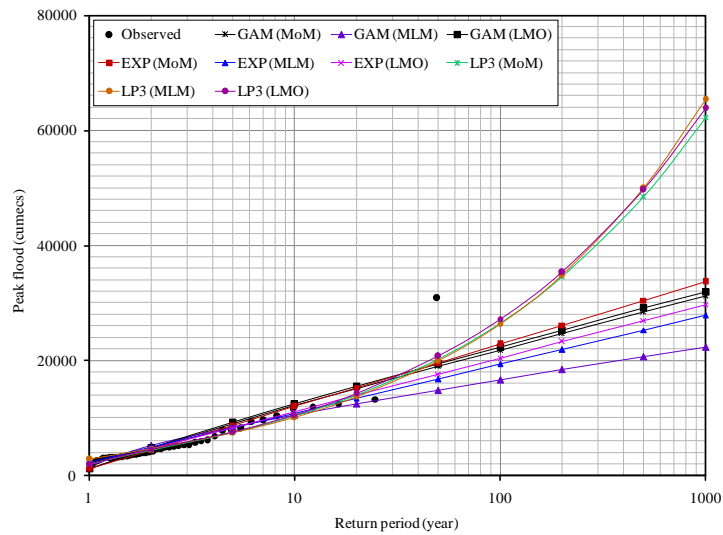


Figure 2: Estimated PF using gamma family of distributions with observed APF

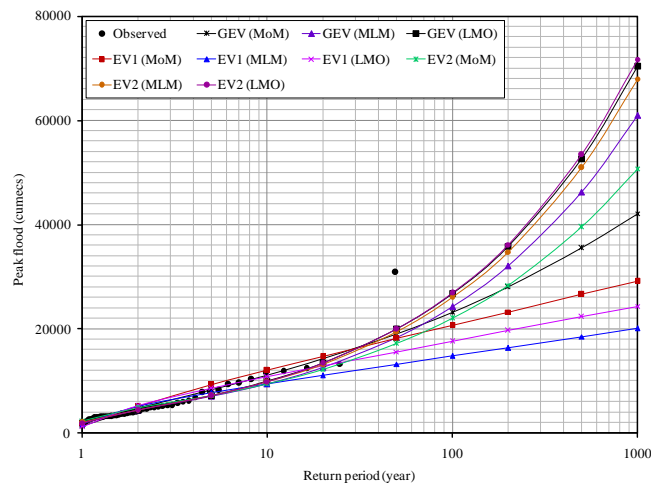


Figure 3: Estimated PF using extreme value family of distributions with observed APF

### *Analysis Based on Diagnostic Test*

The GoF test results showed that both  $\chi^2$  and KS tests results confirmed the applicability of LP3 (using MoM, MLM and LMO), GEV (LMO) and EV2 (MLM and LMO) distributions for FFA for Akhnoor site. However, in addition to GoF tests, for identifying the best suitable probability distribution for estimation of PF, second line of action i.e., D-index was applied, and the D-index values computed for eight probability distributions adopted in FFA are presented in Table 6.

**Table 6: D-index values for Normal, Gamma and Extreme Value Families of distributions**

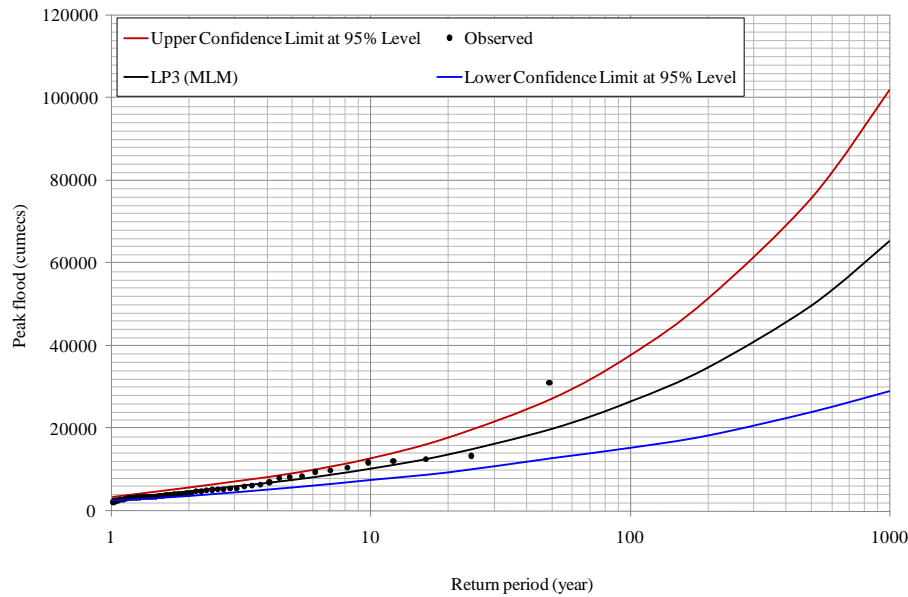
Parameter estimation method	D-index values given by							
	NOR	LN2	GAM	EXP	LP3	GEV	EV1	EV2
MoM	3.134	2.680	3.183	3.070	2.745	2.650	3.122	3.562
MLM	3.091	3.708	3.332	2.861	2.782	3.358	4.638	3.038
LMO	3.857	3.199	3.257	2.751	2.794	2.906	3.011	2.917

From the diagnostic test results, it is noted that:

- i) The D-index values of GEV (MoM), LN2 (MoM), LP3 (MoM), EXP (LMO) and LP3 (MLM) are the first, second, third, fourth and fifth minimum in the order when compared with corresponding values given by MoM, MLM and LMO estimators of other probability distributions.
- ii) But, the PF estimates given by MoM are less accurate when compared to MLM and LMO because of the characteristics of moment estimators, as described earlier.
- iii) Hence, after eliminating the D-index values given by MoM estimators of GEV, LN2 and LP3 distributions, EXP (LMO) is considered as the suitable choice for estimation of PF. But, from Figure 2, it can be seen that most of the observed APF data are lying below the fitted lines of the estimated PF using EXP (LMO).
- iv) In light of the above, LP3 (MLM) is alternatively considered as best fit for estimation of PF. Accordingly, it is suggested that the PF estimates obtained from LP3 (MLM) distribution could be used for planning and design of any hydraulic structures in Akhnoor site.

### *Selection of Probability Distribution*

Based on GoF and diagnostic tests results, it is identified that LP3 (MLM) is better suited for estimation of PF amongst eight probability distributions adopted in FFA. The plots of estimated PF using LP3 (MLM) with 95% confidence limits and observed APF are presented in Figure 4. From which, it can be seen that about 90% of the observed APF are within the confidence limits of the estimated PF using LP3 (MLM) distribution.



**Figure 4: Estimated PF using LP3 (MLM) distribution with 95% confidence limits and observed APF**

## Conclusions

The paper presents the study on comparison of normal, gamma and extreme value families of distributions adopted in FFA. The parameters of the distributions were determined by MoM, MLM and LMO, and are used for estimation of PF at Akhnoor site. The intercomparison of the results was carried out and the following conclusions were drawn from the study:

- i) The estimated PF using EV2 (LMO) distribution are comparatively higher than those values of other probability distributions adopted in FFA for return periods from 100-year and above.
- ii) Qualitative assessment through plots indicated that the fitted lines of the estimated PF using LP3, GEV and EV2 distributions are in the form of exponential curve.
- iii) The  $\chi^2$  test results confirmed the applicability of LP3 (using MoM, MLM and LMO), GEV (LMO) and EV2 (MLM and LMO) distributions for FFA.
- iv) The KS test results supported the use of LN2, EXP, LP3, GEV, EV1 and EV2 distributions for FFA while MoM, MLM and LMO are applied for determining the parameters of the distributions.
- v) Qualitative assessment (plots of FFA results) of the outcomes was weighed with D-index values and accordingly LP3 (MLM) distribution is found to be better suited amongst eight probability distributions studied in FFA for estimation of PF.
- vi) By considering the data length (i.e., 48-years) of APF data used in FFA, the study suggested that the estimated PF beyond 200-year may be cautiously used due to uncertainty in higher order return periods.

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