Advanced Decision Support Systems through Artificial Neural Networks and Data Mining with Correlated Aggregation Operators

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Abstract

This paper addresses Multiple Attribute Group Decision Making (MAGDM) problems where attribute values are expressed as intuitionistic fuzzy numbers. We propose an integrated framework that combines the Choquet integral operator with intuitionistic fuzzy ordered weighted averaging (IFOWA) operators. Unlike conventional models, this approach considers both the significance of individual attributes and their ordered positions, leading to more reliable aggregation. To further enhance efficiency, a data mining algorithm is employed to reduce dimensionality by eliminating redundant or less relevant variables, thereby simplifying the decision process and improving interpretability. The proposed method is validated through a numerical case study, demonstrating improved accuracy and flexibility compared with existing approaches. In addition, Artificial Neural Networks (ANN) are applied to the same problem, offering an alternative computational perspective and confirming the robustness of the framework. Overall, the results highlight the potential of integrating fuzzy aggregation operators, data mining, and ANN techniques to advance the development of intelligent and effective decision support systems.

Keywords: MAGDM, Intuitionistic Fuzzy Sets, Choquet Integral operators, Data Mining, ANN

Introduction

Multiple Attribute Group Decision Making (MAGDM) is the process of selecting the best alternatives. In this competitive world; organization can exist when the correct and appropriate decisions are made. Therefore correct decisions help in successful operation of business by the help of fuzzy sets. Atanassov [1),0] introduced the concept of intuitionistic fuzzy set (IFS) characterized by a membership function and a non-membership function, which is a generalization of the concept of fuzzy set [0]. Data mining applications with statistical methods is inevitable in todays decision making problems. Correlation coefficient of Intuitionistic Fuzzy sets was proposed in [0] which also plays a vital role in data mining in the form of identifying close relationships between fuzzy itemsets. Many aggregation operators which direct the decision making process are found in [0], [0], [0] and [0]. Some induced correlated aggregating operators with intuitionistic fuzzy information and their application to multiple attribute group decision making were proposed in [0]. The aim of this paper is to investigate the multiple attribute decision making problems for evaluating the teaching effectiveness of the higher colleges and universities based on the IFECG operator with intuitionistic fuzzy information and also to apply some data mining techniques for the reduction of the number of alternatives in the decision environment. In [9)] proposed a new framework called the MAGDM-Miner, for mining correlation rules from trapezoidal intuitionistic fuzzy data efficiently. Using this MAGDM-Miner, a decision-maker can overcome the drawbacks in the conventional methods of Decision Support Systems (DSS) especially when dealing with large data-set. In [0],[4)] proposed a novel method for dealing with Decision Making issues that involve Linguistic Intuitionistic Fuzzy (LIF) aspects which are then utilized in the Linguistic Intuitionistic Fuzzy-Technique of Order Preference by Similarity to Ideal Solution (LIF-TOPSIS) method of Decision Support Systems (DSS). In [12)] proposed the Improved Intuitionistic Fuzzy Weighted Arithmetic Averaging (IM-IFWAA) operator and the Improved Intuitionistic Fuzzy Ordered Weighted Averaging (IM-IFOWA) operator. In [0] proposed a suitable decision-making model based on Intuitionistic Fuzzy sets (IFSs) and Gram-Schmidt orthogonalization process for Artificial Neural Network (ANN). In [7)] introduces a new technique for defining the correlation coefficient of triangular and trapezoidal intuitionistic fuzzy sets for solving Multi-attribute Group Decision Making (MAGDM) problems. In [0] was proposed a novel technique of mining Trapezoidal Intuitionistic Fuzzy Correlation Rules for Eigen Valued MAGDM Problems.

Preliminaries

In this section, some basis definitions and IFECG operators are presented.

Definition 1: An IFS A in X is given by and $A=\{\langle x,\mu_A(x),\nu_A(x)\rangle / x\in X\}$ *where* $\mu_A : X\to[0,1]$ and $\nu_A : X\to[0,1]$, with the condition $0\le\mu_A(x)+\nu_A(x)\le 1, \forall x\in X$. The numbers $\mu_A(x)$ and $\nu_A(x)$ represent, respectively, the membership degree and non-membership degree of the element x to the set A.

Definition 2: Let $\tilde{a}=(\mu,\nu)$ be an intuitionistic fuzzy number, a score function S of an intuitionistic fuzzy value can be represented as follows: $S(\tilde{a})=\mu-\nu,\ S(\tilde{a})\in[-1,1]$.

Definition 3: Let $\tilde{a}=(\mu,\nu)$ be an intuitionistic fuzzy number, a accuracy function H of an intuitionistic fuzzy value can be represented as follows: $H(\tilde{a})=\mu+\nu$, $\mathrm{H}(\tilde{a})\in[0,1]$. To evaluate the degree of accuracy of the intuitionistic fuzzy value $\tilde{a}=(\mu,\nu)$, where $H(\tilde{a})\in[0,1]$. The larger the value of $H(\tilde{a})$, the more the degree of accuracy of the intuitionistic fuzzy value $\tilde{a}$.

Definition 4: Let $\tilde{a}=(\mu_j,\nu_j)(j=1,2,...,n)$ be a collection of intuitionistic fuzzy values on Q, and μ be a fuzzy measure on Q, the intuitionistic fuzzy Einstein correlated geometric(IFECG) operator of $\tilde{a}_j$ with respect to μ is defined by:

$$IFECG_\mu(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n)=(\tilde{a}_{\sigma(1)})^{(\mu(A_{(1)})-\mu(A_{(2)}))}\otimes(\tilde{a}_{\sigma(2)})^{(\mu(A_{(2)})-\mu(A_{(3)}))}\otimes...\otimes(\tilde{a}_{\sigma(n)})^{(\mu(A_{(n)})-\mu(A_{(n+1)}))}$$

$$=\bigotimes_{j=1}^{n}(\tilde{a}_{\sigma(j)})^{(\mu(A_{(j)})-\mu(A_{(j+1)}))}$$

$$=\left(\frac{2\prod_{j=1}^{n}\mu_{\sigma(ij)}^{(\mu(A_{(j)})-\mu(A_{(j+1)}))}}{\prod_{j=1}^{n}(2-\mu_{\sigma(ij)})^{(\mu(A_{(j)})-\mu(A_{(j+1)}))}+\prod_{j=1}^{n}\mu_{\sigma(ij)}^{(\mu(A_{(j)})-\mu(A_{(j+1)}))}},\frac{\prod_{j=1}^{n}(1+\nu_{\sigma(ij)})^{(\mu(A_{(j)})-\mu(A_{(j+1)}))}-\prod_{j=1}^{n}(1-\nu_{\sigma(ij)})^{(\mu(A_{(j)})-\mu(A_{(j+1)}))}}{\prod_{j=1}^{n}(1+\nu_{\sigma(ij)})^{(\mu(A_{(j)})-\mu(A_{(j+1)}))}+\prod_{j=1}^{n}(1-\nu_{\sigma(ij)})^{(\mu(A_{(j)})-\mu(A_{(j+1)}))}}\right)$$

Where $(\sigma(1),\sigma(2),...,\sigma(n))$ is a permutation of $(1,2,...,n)$, such that $\tilde{a}_{\sigma(j-1)}\le\tilde{a}_{\sigma(j)}$ for all $j=2,...,n, A_{(i)}=((i),...,(n)), A_{(n+1)}=\phi.$

It can be easily proved that the IFECG operator has the following properties.

Theorem 1: (Idempotency) If all $\tilde{a}_j\ (j=1,2,...,n)$ are equal, i.e. $\tilde{a}_j=\tilde{a}$ for all j, then $IFECG_w(\tilde{a}_1,\tilde{a}_2,...,\tilde{a}_n)=\tilde{a}.$

Theorem 2: (Boundedness) Let $\tilde{a}_j\left(j=1,2,\ldots,n\right)$ be a collection of IFVNs, and let $\tilde{a}^- = \min \tilde{a}_j, \tilde{a}^+ = \max \tilde{a}_j$ Then $\tilde{a}^- \le IFECG_w\left(\tilde{a}_1\tilde{a}_2,\ldots,\tilde{a}_n\right) \le \tilde{a}^+$.

Theorem 3: (Monotonicity) Let $\tilde{a}_j\left(j=1,2,\ldots,n\right)$ and $\tilde{a}'_j\left(j=1,2,\ldots,n\right)$ be two set of IFVNs, if $\tilde{a}_j \le \tilde{a}_j{}'$, for all j ,then $IFECG_w\left(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n\right) \le IFECG_w\left(\tilde{a}_1{}',\tilde{a}_2{}',\ldots,\tilde{a}_n{}'\right)$.

Theorem 4: (commutativity) Let $\tilde{a}_j\left(j=1,2,\ldots,n\right)$ and $\tilde{a}'_j\left(j=1,2,\ldots,n\right)$ be two set of IFVNs, then $IFECG_w\left(\tilde{a}_1,\tilde{a}_2,\ldots,\tilde{a}_n\right) = IFECG_w\left(\tilde{a}_1{}',\tilde{a}_2{}',\ldots,\tilde{a}_n{}'\right)$. Where $\tilde{a}'_j\left(j=1,2,\ldots,n\right)$ is any permutation of $\tilde{a}_j\left(j=1,2,\ldots,n\right)$.

Algorithm For Solving MAGDM Problem

The following assumptions or notations are used to represent the MAGDM problems for evaluating the performance of the sales manager based on the IFECG operator with intuitionistic fuzzy information. Let $A = \{A_1, A_2, \ldots, A_m\}$ be a discrete set of alternatives, $G = \{G_1, G_2, \ldots, G_n\}$ be a set of attributes. The information about attribute weights is completely known. Let $\omega = \{\omega_1, \omega_2, \ldots, \omega_n\}$ be the weight vector of attributes, where $\omega_j \ge 0, j = 1,2\ldots, n$. Suppose that $\tilde{R} = \left(\tilde{r}_{ij}\right)_{m\times n} = \left(\mu_{ij}, \nu_{ij}\right)_{m\times n}$ is the intuitionistic fuzzy decision matrix, where μ_{ij} indicates the degree that the alternative A_i satisfies the attribute G_j given by the decision maker, ν_{ij} indicates the degree that the alternative A_i doesn't satisfy the attribute G_j given by the decision maker,

$$\mu_{ij} \subset [0,1], \nu_{ij} \subset [0,1], \mu_{ij} + \nu_{ij} \le 1, i = 1,2,\ldots, m, j = 1,2,\ldots, n.$$

In the following, we apply the IFECG operator to multiple attribute decision making (MADM) problems for evaluating the performance of the sales managers based on the IFECG operator with intuitionistic fuzzy information.

Step 1: Determine the fuzzy measure of attribute of $G_j\left(j=1,2,\ldots,n\right)$ and attribute sets of G. There are a few methods for the determination of the fuzzy measure. For example, linear methods, quadratic methods, heuristic-based methods and genetic algorithms and so on are available in the literature.

Step 2: Utilize the IFCGM operator

$\tilde{r}_i = IFECG(\tilde{r}_{i1}, \tilde{r}_{i2}, \ldots, \tilde{r}_{in}) = (\tilde{r}_{\sigma(i1)})^{(\mu(G_{(1)})-\mu(G_{(2)}))} \otimes (\tilde{r}_{\sigma(i2)})^{(\mu(G_{(2)})-\mu(G_{(3)}))} \otimes \ldots \otimes (\tilde{r}_{\sigma(in)})^{(\mu(G_{(n)})-\mu(G_{(n+1)}))} = \bigotimes_{j=1}^{n} (\tilde{r}_{(ij)})^{(\mu(G_{(j)})-\mu(G_{(j+1)}))}$

to derive the overall preference values $\tilde{r}_i \left(i = 1, 2, \ldots, m\right)$ of the alternative A_i.

Step 3: Calculate the scores $S\left(\tilde{r}_i\right)\left(i = 1, 2, \ldots, m\right)$ of the collective overall intuitionistic fuzzy preference values $\tilde{r}_i \left(i = 1, 2, \ldots, m\right)$ to rank all the alternatives $A_i \left(i = 1, 2, \ldots, m\right)$ and then to select the best one(s).

Step 4: Rank all the enterprises $A_i \left(i = 1, 2, \ldots, m\right)$ and select the best one(s) in accordance with $S\left(\tilde{r}_i\right)$ and $\mathrm{H}\left(\tilde{r}_i\right)\left(i = 1, 2, \ldots, m\right)$.

Step 5: The fuzzy support fsupp $(\tilde{r}_i)$, of each trapezoidal IFS item is computed.

(5.1) Let $L_1 = \left\{\tilde{r}_p / \tilde{r}_p \in F,\ \text{fsupp}(\tilde{r}_p) \geq s_{\tilde{r}}\right\}$ be the set of frequent fuzzy item sets whose size is equal to 1.

(5.2) Let $C_2 = \left\{(F_A, F_B)\right\}$ be the set of all combinations of two elements belonging to L_1, where $F_A, F_B \in L_1, F_A \neq F_B$. That is, $C_2 = \left\{(F_A, F_B)\right\}$ is generated by L_1 joint with L_1. Since F_A, F_B are the elements of L_1, the size of each element of C_2 is 2.

(5.3) For each element of C_2, (F_A, F_B), the fuzzy support, fsupp$\{(F_A, F_B)\}$ is computed by using the comparison of the hesitation degree of each intuitionistic fuzzy information, and then the trapezoidal intuitionistic fuzzy correlation coefficient between F_A, F_B , $K(F_A, F_B)$ is computed from equation (11). Calculate the Median value $K_{MED}(F_A, F_B)$ ofall the correlation coefficient $K(F_A, F_B)$ and consider all the $K(F_A, F_B) > K_{MED}(F_A, F_B)$ for the next level.

(5.4) For each element, whose fuzzy support is greater than or equal to $s_{\tilde{r}}$ and the maximum fuzzy correlation coefficients with their correlation coefficient $K(F_A, F_B) > K_{MED}(F_A, F_B)$ of C_2, will be an element of L_2. Hence, L_2 is the set of the frequent combinations of two fuzzy itemsets, and the size of each element being 2.

(5.5) Next, each C_k, $k \geq 3$, is generated by L_{k-1} joint with L_{k-1}. Assume that (F_W, F_X) and (F_Y, F_Z) are two elements of L_{k-1}, where $F_X = F_Y$. If the size of the combination $(F_X, \{F_W, F_Z\})$ is k, and (F_W, F_Z) is also a frequent combination of 2-fuzzy itemsets, then the combination $(F_X, \{F_W, F_Z\})$ is an element with size k of C_k. For each element of C_k, its fuzzy support and fuzzy correlation coefficient are still used to find the elements of L_k.

(5.6) When each L_k, $k \geq 2$, is obtained, for each element of L_k, (F_G, F_H), the 2-candidate fuzzy correlation rules can be generated. At this level, the itemsets with the highest correlation coefficient can be selected or ranked, which can be considered as interesting fuzzy correlation rules.

The loop will stop when next C_{k+1} cannot be generated.

Numerical Example

To demonstrate the approach proposed in this study, a numerical example is provided in this section. Suppose a retail company wants to evaluate the performance of its sales managers. Five potential sales managers are under consideration for promotion. To assess them, the company selects four attributes:

- **G1:** Customer feedback
- **G2:** Peer evaluation
- **G3:** Expert/consultant evaluation
- **G4:** Leadership and management skills

Using the following decision matrix, the decision maker evaluates the five candidates under these four criteria, with the evaluations expressed in intuitionistic fuzzy information.

$$A = \begin{bmatrix} (0.3,0.4) & (0.4,0.6) & (0.8,0.1) & (0.6,0.3) \\ (0.7,0.1) & (0.5,0.3) & (0.6,0.2) & (0.4,0.3) \\ (0.3,0.6) & (0.3,0.4) & (0.5,0.4) & (0.6,0.3) \\ (0.7,0.2) & (0.4,0.5) & (0.6,0.3) & (0.7,0.3) \\ (0.5,0.3) & (0.9,0.1) & (0.6,0.3) & (0.5,0.4) \end{bmatrix}.$$

Then, we utilize the approach developed to evaluate performance of the sales managers.

Step 1. Suppose the fuzzy measure of attribute of $G_j\left(j=1,2,3,4\right)$ and attribute sets of G as follows:

$$\mu\left(G_1\right)=0.30,\ \mu\left(G_2\right)=0.20,\ \mu\left(G_3\right)=0.35,\ \mu\left(G_4\right)=0.25,\ \mu\left(G_1,G_2\right)=0.50,$$
$$\mu\left(G_1,G_3\right)=0.65,\ \mu\left(G_1,G_4\right)=0.70,\ \mu\left(G_2,G_3\right)=0.65,\ \mu\left(G_2,G_4\right)=0.60,$$
$$\mu\left(G_3,G_4\right)=0.50,\ \mu\left(G_1,G_2,G_3\right)=0.80,\ \mu\left(G_1,G_2,G_4\right)=0.85,$$
$$\mu\left(G_1,G_3,G_4\right)=0.85,\ \mu\left(G_2,G_3,G_4\right)=0.78,\ \mu\left(G_1,G_2,G_3,G_4\right)=1.00$$

Step 2. Utilize the decision information given in matrix $\tilde{R}$, and the IFECG operator, we obtain the overall preference values $\tilde{r}_i$ of the performance of the sales managers $A_i\left(i=1,2,...,5\right)$.

$$\tilde{r}_i=\left(\frac{2\prod_{j=1}^{n}\mu_{\sigma(ij)}^{\left(\mu\left(A_{(j)}\right)-\mu\left(A_{(j+1)}\right)\right)}}{\prod_{j=1}^{n}\left(2-\mu_{\sigma(ij)}\right)^{\left(\mu\left(A_{(j)}\right)-\mu\left(A_{(j+1)}\right)\right)}+\prod_{j=1}^{n}\mu_{\sigma(ij)}^{\left(\mu\left(A_{(j)}\right)-\mu\left(A_{(j+1)}\right)\right)}},\frac{\prod_{j=1}^{n}\left(1+v_{\sigma(ij)}\right)^{\left(\mu\left(A_{(j)}\right)-\mu\left(A_{(j+1)}\right)\right)}-\prod_{j=1}^{n}\left(1-v_{\sigma(ij)}\right)^{\left(\mu\left(A_{(j)}\right)-\mu\left(A_{(j+1)}\right)\right)}}{\prod_{j=1}^{n}\left(1+v_{\sigma(ij)}\right)^{\left(\mu\left(A_{(j)}\right)-\mu\left(A_{(j+1)}\right)\right)}+\prod_{j=1}^{n}\left(1-v_{\sigma(ij)}\right)^{\left(\mu\left(A_{(j)}\right)-\mu\left(A_{(j+1)}\right)\right)}}\right)$$

For i=1,

$$\tilde{r}_1=\left(\frac{2(0.3)^{0.30}\times(0.4)^{0.20}\times(0.8)^{0.30}\times(0.6)^{0.20}}{(2-0.3)^{0.30}\times(2-0.4)^{0.20}\times(2-0.8)^{0.30}\times(2-0.6)^{0.20}+(0.3)^{0.30}\times(0.4)^{0.20}\times(0.8)^{0.30}\times(0.6)^{0.20}},\right.$$
$$\left.\frac{(1+0.4)^{0.30}\times(1+0.6)^{0.20}\times(1+0.1)^{0.30}\times(1+0.3)^{0.20}-(1-0.4)^{0.30}\times(1-0.6)^{0.20}\times(1-0.1)^{0.30}\times(1-0.3)^{0.20}}{(1+0.4)^{0.30}\times(1+0.6)^{0.20}\times(1+0.1)^{0.30}\times(1+0.3)^{0.20}+(1-0.4)^{0.30}\times(1-0.6)^{0.20}\times(1-0.1)^{0.30}\times(1-0.3)^{0.20}}\right)$$

$$\tilde{r}_1=\left(0.5039,0.344\right)$$

For i=2,

$$\tilde{r}_2=\left(\frac{2(0.7)^{0.30}\times(0.5)^{0.20}\times(0.6)^{0.30}\times(0.4)^{0.20}}{(2-0.7)^{0.30}\times(2-0.5)^{0.20}\times(2-0.6)^{0.30}\times(2-0.4)^{0.20}+(0.7)^{0.30}\times(0.5)^{0.20}\times(0.6)^{0.30}\times(0.4)^{0.20}},\right.$$
$$\left.\frac{(1+0.1)^{0.30}\times(1+0.3)^{0.20}\times(1+0.2)^{0.30}\times(1+0.3)^{0.20}-(1-0.1)^{0.30}\times(1-0.3)^{0.20}\times(1-0.2)^{0.30}\times(1-0.3)^{0.20}}{(1+0.1)^{0.30}\times(1+0.3)^{0.20}\times(1+0.2)^{0.30}\times(1+0.3)^{0.20}+(1-0.1)^{0.30}\times(1-0.3)^{0.20}\times(1-0.2)^{0.30}\times(1-0.3)^{0.20}}\right)$$

$$\tilde{r}_2=\left(0.5630,0.2115\right)$$

For i=3,

$$\tilde{r}_3=\left(\frac{2(0.7)^{0.30}\times(0.5)^{0.20}\times(0.6)^{0.30}\times(0.4)^{0.20}}{(2-0.7)^{0.30}\times(2-0.5)^{0.20}\times(2-0.6)^{0.30}\times(2-0.4)^{0.20}+(0.7)^{0.30}\times(0.5)^{0.20}\times(0.6)^{0.30}\times(0.4)^{0.20}},\right.$$
$$\left.\frac{(1+0.1)^{0.30}\times(1+0.3)^{0.20}\times(1+0.2)^{0.30}\times(1+0.3)^{0.20}-(1-0.1)^{0.30}\times(1-0.3)^{0.20}\times(1-0.2)^{0.30}\times(1-0.3)^{0.20}}{(1+0.1)^{0.30}\times(1+0.3)^{0.20}\times(1+0.2)^{0.30}\times(1+0.3)^{0.20}+(1-0.1)^{0.30}\times(1-0.3)^{0.20}\times(1-0.2)^{0.30}\times(1-0.3)^{0.20}}\right)$$

$$\tilde{r}_3=\left(0.3895,0.4476\right)$$

For i=4,

$$\tilde{r}_4 = \left(\frac{2(0.7)^{0.30} \times (0.4)^{0.20} \times (0.6)^{0.30} \times (0.7)^{0.20}}{(2-0.7)^{0.30} \times (2-0.4)^{0.20} \times (2-0.6)^{0.30} \times (2-0.7)^{0.20} + (0.7)^{0.30} \times (0.4)^{0.20} \times (0.6)^{0.30} \times (0.7)^{0.20}}, \right.$$

$$\left. \frac{(1+0.2)^{0.30} \times (1+0.5)^{0.20} \times (1+0.3)^{0.30} \times (1+0.3)^{0.20} - (1-0.2)^{0.30} \times (1-0.5)^{0.20} \times (1-0.3)^{0.30} \times (1-0.3)^{0.20}}{(1+0.2)^{0.30} \times (1+0.5)^{0.20} \times (1+0.3)^{0.30} \times (1+0.3)^{0.20} + (1-0.2)^{0.30} \times (1-0.5)^{0.20} \times (1-0.3)^{0.30} \times (1-0.3)^{0.20}} \right)$$

$$\tilde{r}_4 = (0.6027, 0.3144)$$

For i=5,

$$\tilde{r}_5 = \left(\frac{2(0.5)^{0.30} \times (0.9)^{0.20} \times (0.6)^{0.30} \times (0.5)^{0.20}}{(2-0.5)^{0.30} \times (2-0.9)^{0.20} \times (2-0.6)^{0.30} \times (2-0.5)^{0.20} + (0.5)^{0.30} \times (0.9)^{0.20} \times (0.6)^{0.30} \times (0.5)^{0.20}}, \right.$$

$$\left. \frac{(1+0.3)^{0.30} \times (1+0.1)^{0.20} \times (1+0.3)^{0.30} \times (1+0.4)^{0.20} - (1-0.3)^{0.30} \times (1-0.1)^{0.20} \times (1-0.3)^{0.30} \times (1-0.4)^{0.20}}{(1+0.3)^{0.30} \times (1+0.1)^{0.20} \times (1+0.3)^{0.30} \times (1+0.4)^{0.20} + (1-0.3)^{0.30} \times (1-0.1)^{0.20} \times (1-0.3)^{0.30} \times (1-0.4)^{0.20}} \right)$$

$$\tilde{r}_5 = (0.3540, 0.2513)$$

Step 3. Calculate the scores $S(\tilde{r}_i)(i = 1, 2, 3, 4, 5)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i (i = 1, 2, 3, 4, 5)$.

$S(\tilde{r}_1) = 0.1599, S(\tilde{r}_2) = 0.3515, S(\tilde{r}_3) = -0.0581, S(\tilde{r}_4) = 0.2883, S(\tilde{r}_5) = 0.2817.$

Step 4. Rank all the performance of the sales managers $A_i (i = 1, 2, 3, 4, 5)$ in accordance with the scores $S(\tilde{r}_i)(i = 1, 2, 3, 4, 5)$ of the overall intuitionistic fuzzy preference values $\tilde{r}_i (i = 1, 2, 3, 4, 5): A_2 \succ A_4 \succ A_5 \succ A_1 \succ A_3$, and thus the most desirable performance of the sales managers is A_2 .

Step 5: Using the above data mining algorithm, we are able to reduce the number of decision variables as follows $A_2 \succ A_4 \succ A_5$. A_1 and A_3 are eliminated from the system of decision making process.

Pseudo-Code for Perceptron Based Ranking Method

1. **Input Preparation**
    - Collect the defuzzified decision matrix $X = \{x_1, x_2, \dots, x_m\}$, where each x_i is an n-dimensional vector of attributes.
    - Define the target output labels $y = \{y_1, y_2, \dots, y_m\}$.
    - Set learning rate η and maximum number of epochs E.

Symbols:

$$X \in \mathbb{R}^{m \times n}, y \in \{0,1\}^m$$

2. Initialization

- Start with weights set to zero: $w = [0,0, \ldots ,0]$.
- Initialize the bias term: $b = 0$.

Symbols:

$$w \leftarrow 0, b \leftarrow 0$$

3. Training phase

- Repeat for each epoch until convergence or maximum *E*:

  a. For each alternative x_i:

  - Compute the activation value:

$$z_i = w \cdot x_i + b$$

  - Assign the predicted class:

$$\hat{y}_i = \begin{cases} 1 & \text{if } z_i > 0 \\ 0 & \text{otherwise} \end{cases}$$

  - Compute the error:

$$e_i = y_i - \hat{y}_i$$

  - If $e_i \neq 0$, updated the parameters:

$$w \leftarrow w + \eta e_i x_i, b \leftarrow b + \eta e_i$$

  b. If no errors are found in the entire epoch, stop training (model has converged.)

4. Testing and scoring

- After training, compute a score for each alternative using the learned weights and bias:

$$s_i = w \cdot x_i + b$$

- A higher score corresponds to a stronger preference for that alternative.

5. Ranking

- Sort the alternatives in descending order of s_i.
- Output the ranked list, where the first element is the best alternative.

Symbols:

$$\text{Rank}(X) = \text{argsort}(-s_i), i = 1, \ldots., m$$

Output:

Training converged at epoch 5

Final Weights: [0.07, -0.08, 0.01, 0.06]

Final Bias: 0.0

Results:

A1: score=0.0280, predicted=1

A2: score=0.0180, predicted=1

A3: score=-0.0100, predicted=0

A4: score=0.0000, predicted=0

A5: score=0.0210, predicted=1

Ranking of Alternatives:

1) A1 (score=0.0280)

2) A5 (score=0.0210)

3) A2 (score=0.0180)

4) A4 (score=0.0000)

5) A3 (score=-0.0100)

From the above we can observe that A1 is the best alternative among all the available ones. Hence, the first person is preferred to be the sales manager.

Conclusion

The study demonstrates that integrating the Choquet integral with IFOWA operators, supported by data mining techniques, provides a more accurate and interpretable solution to MAGDM problems under intuitionistic fuzzy environments. The application of ANN further validates the robustness of the proposed framework, offering an additional computational perspective. Overall, the findings confirm that the combined use of fuzzy aggregation, dimensionality reduction, and ANN can significantly enhance the effectiveness and intelligence of modern decision support systems.

Author Contributions

Conceptualization, Karpaha C and John Robinson P; Methodology, Karpaha C; Software, John Robinson P; Validation, Karpaha C and John Robinson P; formal analysis, Karpaha C; investigation, John Robinson P; resources, Karpaha C; data maintenance, John Robinson P; writing-creating the initial design, Karpaha C; writing-reviewing and editing, Karpaha C; visualization, John Robinson P; monitoring, John Robinson P; project management, John Robinson P. All authors have read and agreed to the published version of the manuscript. Authorship must be limited to those who have made a significant contribution to the work reported.

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Conflicts of Interest

This is the original work of the authors and all authors have seen and approved the final version of the manuscript being submitted. The material described here is not under publication or consideration for publication elsewhere. “The authors declare no conflict of interest.

References

1) *Atanassov, K. (1986). Intuitionistic fuzzy sets. Fuzzy Sets and Systems, 20(1), 87–96.*

2) *Atanassov, K. (1989). More on intuitionistic fuzzy sets. Fuzzy Sets and Systems, 33(1), 37–45.*

3) *Leonishiya, A., & Robinson, J. (2023a). Varieties of linguistic intuitionistic fuzzy distance measures for linguistic intuitionistic fuzzy TOPSIS method. Indian J. Sci. Technol, 16(33), 2653-2662.*

4) *Leonishiya, A., & Robinson, P. J. (2023b). A Fully Linguistic Intuitionistic Fuzzy Artificial Neural Network Model for Decision Support Systems. Indian J. Sci. Technol, 16, 29-36.*

5) *Nirmalsingh, S. S., & Robinson, P. J. (2023). Triangular Intuitionistic Fuzzy Linear Programming Problem with a New Ranking Method based on Correlation Coefficient. Indian Journal of Science and Technology, 16(3), 75-83.*

6) *Robinson, J.P., & Amirtharaj, E. H. (2011). A short primer on the Correlation coefficient of Vague sets. International Journal of Fuzzy System Applications (IJFSA), 1(2), 55-69.*

7) *Robinson, J.P., & Amirtharaj, E. H. (2012). A search for the correlation coefficient of triangular and trapezoidal intuitionistic fuzzy sets for multiple attribute group decision making. In Mathematical Modelling and Scientific Computation: International Conference, ICMMSC 2012, Gandhigram, Tamil Nadu, India, March 16-18, 2012. Proceedings (pp. 333-342). Springer Berlin Heidelberg.*

8) *Robinson, J.P., & Amirtharaj, E. H. (2014). Efficient multiple attribute group decision making models with correlation coefficient of vague sets. International Journal of Operations Research and Information Systems, 5(3), 27-49.*

9) *Robinson, J. P., & Amirtharaj, H. (2014). MAGDM-miner: a new algorithm for mining trapezoidal intuitionistic fuzzy correlation rules. International Journal of Decision Support System Technology (IJDSST), 6(1), 34-59.*

10) *Robinson, J.P., & Jeeva, S., (2016). Mining Trapezoidal Intuitionistic Fuzzy Correlation Rules for Eigen Valued MAGDM Problems. International Journal of Control Theory and Applications, 9(7), 585-616.*

11) *Robinson, J., & EC, H. A. (2017). Vague Correlation Coefficient of Interval Vague Sets and its Applications to Topsis in MADM Problems. In Fuzzy Systems: Concepts, Methodologies, Tools, and Applications (pp. 1110-1149). IGI Global Scientific Publishing.*

12) *Robinson, P. J., & Prakash, P. W. A. (2024). Application of Improved Class of Weighted Averaging Operators in Intuitionistic Fuzzy Ann Guided MAGDM E-Commerce Problems. Indian Journal of Science and Technology, 17(29), 3026-3036.*

13) *Robinson, P. J., & Saranraj, A. (2024). Intuitionistic Fuzzy Gram-Schmidt Orthogonalized Artificial Neural Network for Solving MAGDM Problems. Indian J. Sci. Technol, 17, 2529-2537.*

14) *Wei, G. (2009). "Some geometric aggregation functions and their application to dynamic multiple attribute decision making in intuitionistic fuzzy setting", International Journal of Uncertainty, Fuzziness and Knowledge- Based Systems, 17(2), 179-196.*

15) *Wei, G., & Zhao, X. (2012). Some induced correlated aggregating operators with intuitionistic fuzzy information and their application to multiple attribute group decision making. Expert Systems with Applications, 39(2), 2026–2034.*

16) *Xu, Z., & Yager, R.R. (2008). Dynamic intuitionistic fuzzy multi-attribute decision making, International Journal of Approximate Reasoning, 48(1), 246-262.*

17) *Xu, Z., & Yager, R.R. (2011). Intuitionistic Fuzzy Bonferroni Means, IEEE Transactions on Systems Man and Cybernetics Part B-Cybernetics, 41, 568-578.*

18) *Zadeh, L. A. (1965). Fuzzy Sets. Information and Control, 8(3), 338–356.*

19) *Zeng, W., & Li, H. (2007). Correlation Coefficient of Intuitionistic Fuzzy sets. Journal of Industrial Engineering International, 3, 33–40.*