Solving MAGDM Problems with Intuitionistic Fuzzy Linguistic Artificial Neural Networks

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Abstract

This work introduces the LinIF-ANN (Linguistic Intuitionistic Fuzzy Artificial Neural Network), an innovative framework for Multi-Attribute Group Decision-Making (MAGDM) problems in commerce and business analytics. The model extends Linguistic Intuitionistic Fuzzy Sets (LIFS) to capture hesitation and uncertainty inherent in qualitative decision contexts. Unlike traditional aggregation-based methods, LinIF-ANN integrates a perceptron-based neural network with linguistic intuitionistic fuzzy inputs, allowing dynamic learning and adjustment of decision weights for improved ranking accuracy. To enhance aggregation, a novel Linguistic Intuitionistic Fuzzy Weighted Arithmetic Aggregation (Lin-IFWAA) operator is proposed, complemented by new defuzzification strategies tailored for linguistic data. The Delta learning rule, adapted for LIFS, ensures effective processing of complex decision inputs. Empirical results demonstrate that the framework not only enhances decision quality but also delivers interpretable, scalable, and reliable solutions, making it particularly suitable for commerce-oriented decision support systems.

Keywords: Linguistic Intuitionistic Fuzzy Sets, ANN, MAGDM, Delta Learning, Decision Support Systems, Commerce Analytics

1|Introduction

The Artificial Neural Network (ANN) has its wide range of applications in various fields and a few to mention are image processing, speech recognition, machine learning and in many medical diagnoses. ANN is also one of the DSS which operates with the idea of ranking the best alternative out of the available ones in any decision system whenever attributes with difference of opinion are involved [1], [2], [4], [5]. The Multiple Attribute Group Decision Making (MAGDM) problems also operates in the same way as the ANN and the difference between them is the choice of the processing functions and the ranking functions. Decision making methods like TOPSIS method is where the decision making problem will concentrate its methodology based on the ranking methods done by measuring the closeness to the positive or negative ideal solution [6],[8] and [12]. In recent days, linguistic intuitionistic fuzzy data has gained the attention of researchers to a large extent [3], [6], [7], [9] and [13]. There are various aggregation operators proposed by researchers where a few can be mentioned as cited in this article. The authors in [13] expanded TOPSIS technique for MAGDM problems and it is obvious that the Fuzzy metric and distance measures are extremely important in Fuzzy Decision Making situations. Intuitionistic Fuzzy Neural Network was the focus of the authors in [1], [2], [3], [4], [6], [7], [12] and [14]. In this work we have proposed a new algorithm for Linguistic Intuitionistic Fuzzy ANN (LIF-ANN) and based on attribute weight determination and also ranking of the alternatives. As a part of aggregation process, a new operator named Linguistic Intuitionistic Fuzzy Ordered Weighted Arithmetic Aggregation (Lin-IFWAA) operator is proposed and some theorems are proved for the operator. Different computations are performed with the proposed new algorithm of ANN and comparisons are made with utilizing crisp variables for the final ranking of the best alternatives in the decision algorithms.

2|Preliminaries

Robinson and Leonishiya [10] previously introduced refined operations on Linguistic Intuitionistic Fuzzy Sets (LIFS) aimed at improving multi-attribute group decision-making under uncertainty. Their approach integrated linguistic terms—such as “very high,” “moderate,” and “low”—with intuitionistic fuzzy parameters: membership (μ), non-membership (ν), and hesitation (π). They proposed novel aggregation mechanisms, including linguistic IF-weighted averaging and comparative normalization strategies, which enhanced the interpretability and consistency of decision outcomes. This framework laid the groundwork for more intuitive human-centric models in MAGDM environments characterized by imprecise and linguistically expressed data. Here we shall discuss some of the basic operations pertaining LIFS.

2.1|Definition

Let $\tilde{\sigma}_1 = < l_{\theta(\sigma_1)}, (\alpha(\sigma_1), \gamma(\sigma_1)) >$ and $\tilde{\sigma}_2 = < l_{\theta(\sigma_2)}, (\alpha(\sigma_2), \gamma(\sigma_2)) >$ be two LIFNs and $\lambda > 0$. Then the operations of LIFNs are defined as:

$\tilde{\sigma}_1+\tilde{\sigma}_2=<l_{\theta(\sigma_1)+\theta(\sigma_2)}, (\alpha(\sigma_1)+\alpha(\sigma_2)-\alpha(\sigma_1)\alpha(\sigma_2),\gamma(\sigma_1)\gamma(\sigma_2))>$,

$\tilde{\sigma}_1 \otimes \tilde{\sigma}_2=<l_{\theta(\sigma_1)\times\theta(\sigma_2)}, (\alpha(\sigma_1)\alpha(\sigma_2),\gamma(\sigma_1)+\gamma(\sigma_2)-\gamma(\sigma_1)\gamma(\sigma_2))>$,

$\lambda\tilde{\sigma}_1=<l_{\lambda\times\theta(\sigma_1)}, (1-(1-\alpha(\sigma_1))^{\lambda}, (\gamma(\sigma_1))^{\lambda})>$, and

$\tilde{\sigma}_1{}^{\lambda}=<l_{\theta(\sigma_1)^{\lambda}}, (\alpha(\sigma_1))^{\lambda},1-(1-(\gamma(\sigma_1))^{\lambda})>$. Where $\alpha(\sigma_1),\gamma(\sigma_1)$ and $\theta(\sigma_1)$ denotes membership, non-membership and linguistic degree of given LIFN.

3|A New Attribute Weight Determination Method and Aggregation Operator for ANN

A. Linguistic Median Membership (LMM) function

Let $\tilde{\sigma}_1=<l_{\theta(\sigma_1)}, (\alpha(\sigma_1),\gamma(\sigma_1))>$ for j=1,2,…,n be a collection of Linguistic Intuitionistic fuzzy numbers. Then the Linguistic Median Membership function is defined as $M_m = \frac{(\theta(x)+(\alpha_A(x)+1-\gamma_A(x))}{2}$. It is used to defuzzify the LIFN.

B. The LinIFWAA Operator

Let $\tilde{\sigma}_1=<l_{\theta(\sigma_1)}, (\alpha(\sigma_1),\gamma(\sigma_1))>$ for j=1,2,…,n be a collection of Linguistic Intuitionistic fuzzy numbers. The Linguistic Intuitionistic Fuzzy Weighted Arithmetic Averaging (LinIFWAA) operator LinIFWAA: $Q^n \to Q$ is defined as:

$$\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n)=r_{ij}=\left(S_{\prod_{j=1}^{n}\theta_j w_j}, 1-\prod_{j=1}^{n}(1-\alpha_j)^{w_j}, \prod_{j=1}^{n}\gamma_j^{w_j}\right)$$

where, $w = (w_1, w_2,....,w_n)^T$ is the weight vector of $\tilde{\sigma}_j$, j=1,2,…,n and for $w_j>0$, $\sum_{j=1}^{n} w_j =1$.

C. Theorem 1

Let $\tilde{\sigma}_j$, (j=1,2,.....,n), be a collection of Linguistic Intuitionistic Fuzzy numbers; then the aggregated value by using the *LinIFWAA* operator is also a Linguistic Intuitionistic Fuzzy number and

$$\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n)=r_{ij}$$

$$=\left(S_{\prod_{j=1}^{n}\theta_j w_j}, 1-\prod_{j=1}^{n}(1-\alpha_j)^{w_j}, \prod_{j=1}^{n}\gamma_j^{w_j}\right)$$

where $w = (w_1,w_2,...,w_n)^T$ is the weight vector of the *LinIFWAA* operator with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j =1$.

Proof: Let us prove this by the method of induction. Let us consider $\tilde{\sigma}_{\sigma(1)}w_1$ and $\tilde{\sigma}_{\sigma(2)}w_2$.

$$\tilde{\sigma}_1 w_1 = \langle S_{\theta_1 w_1}, (1-[1-\alpha_1]^{w_1}, [\gamma_1]^{w_1})\rangle, \tilde{\sigma}_2 w_2 = \langle S_{\theta_2 w_2}, (1-[1-\alpha_2]^{w_2}, [\gamma_2]^{w_2})\rangle.$$

Then $\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2) = \tilde{\sigma}_1 w_1 \oplus \tilde{\sigma}_2 w_2 = \langle S_{\theta_1 w_1+\theta_2 w_2}, [1-[1-\alpha_1]^{w_1}[1-\alpha_2]^{w_2}, [\gamma_1]^{w_1}[\gamma_2]^{w_2}]\rangle$,

Continuing the process with $\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_k$ we have :

$$\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_k)=\left\langle S_{\prod_{j=1}^{k}\theta_j w_j}, \left[1-\prod_{j=1}^{k}(1-\alpha_j)^{w_j}, \prod_{j=1}^{k}\gamma_j^{w_j}\right]\right\rangle.$$

Then when n=k+1 we have:

$$\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_k,\tilde{\sigma}_{k+1})=\left\langle S_{\prod_{j=1}^{k+1}\theta_j w_j}, \left[1-\prod_{j=1}^{k+1}(1-\alpha_j)^{w_j}, \prod_{j=1}^{k+1}\gamma_j^{w_j}\right]\right\rangle.$$ Hence, we see that the operator holds for n=k+1. Then by the principle of induction, the operator is true for all n, which completes the proof. Hence,

$$\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n)=r_{ij}$$

$$=\left(S_{\prod_{j=1}^{n}\theta_j w_j}, 1-\prod_{j=1}^{n}(1-\alpha_j)^{w_j}, \prod_{j=1}^{n}\gamma_j^{w_j}\right).$$

D. Theorem 2

Let $\tilde{\sigma}_j$, (j=1,2,.....,n), be a collection of Linguistic Intuitionistic Fuzzy numbers and $w = (w_1,w_2,...,w_n)^T$ be the weight vector of $\tilde{\sigma}_j$, with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$.Then we prove that the *LinIFWAA* operator is (i) Idempotent, (ii)Bounded,(iii) Monotonic and (iv) Commutative.

Proof

(i) Idempotent: If all $\tilde{\sigma}_j$, (j=1,2,.....,n), are equal, ie.$\tilde{\sigma}_j = \tilde{\sigma}$ for all j, then we have to prove $\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_k) = \tilde{\sigma}$.Since $\tilde{\sigma}_j = \tilde{\sigma}$, for all j, then we should have,

$$\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n) = \sum_{j=1}^{n} \tilde{a}\omega_j$$

$$= \left\langle \sum_{j=1}^{n} \theta_j\omega_j, \left[1-\prod_{j=1}^{n}(1-\alpha_j)^{w_j}, \prod_{j=1}^{n}\gamma_j^{w_j}\right]\right\rangle$$

$$= \left\langle S_{\theta_j}, [\alpha_{\tilde{\sigma}_j}], [\gamma_{\tilde{\sigma}_j}]\right\rangle = \tilde{\sigma}.$$

(ii) Boundedness: Let, $\tilde{\sigma}^- \leq \text{LinIFWAA}_\omega(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n) \leq \tilde{\sigma}^+$.

This is evident from the following: $\tilde{\sigma}^- = \left\langle \min_i S_{\theta_i}; \left[\min_i \alpha_{\tilde{\sigma}_i}\right], \left[\max_i \gamma_{\tilde{\sigma}_i}\right]\right\rangle$.

(iii) Monotonicity: Let $\tilde{\sigma}_j^*$, (j=1,2,.....,n), be a collection of Linguistic Intuitionistic Fuzzy numbers. If$\tilde{\sigma}_j \le \tilde{\sigma}_j^*$ for all j, then we, prove that

$\text{LinIFWAA}_\omega(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n) \le \text{LinIFWAA}_\omega(\tilde{\sigma}_1{}^*,\tilde{\sigma}_2{}^*,....,\tilde{\sigma}_n{}^*)$for all ω.

Let, $\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n) = \sum_{j=1}^{n} \tilde{\sigma}_i\, w_i$&$\text{LinIFWAA}_w(\tilde{\sigma}_1{}^*,\tilde{\sigma}_2{}^*,....,\tilde{\sigma}_n{}^*) = \sum_{j=1}^{n} \tilde{\sigma}_i{}^*\, w_i$.

Since $\tilde{\sigma}_j \le \tilde{\sigma}_j^*$ for all j, then we have

$\text{LinIFWAA}_\omega(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n) \le \text{LinIFWAA}_\omega(\tilde{\sigma}_1{}^*,\tilde{\sigma}_2{}^*,....,\tilde{\sigma}_n{}^*)$.

This is evident from the fact when

$\tilde{\sigma}_j = \left\langle S_{\theta_j}, [\alpha_{\tilde{\sigma}_j}], [\gamma_{\tilde{\sigma}_j}] \right\rangle$ and $\tilde{\sigma}^*{}_j = \left\langle S_{\theta_j{}^*}, [\alpha_{\tilde{\sigma}_j{}^*}], [\gamma_{\tilde{\sigma}_j{}^*}] \right\rangle$, we should have, $S_{\theta_j} \le S_{\theta_j{}^*}, \alpha_{\tilde{\sigma}_j} \le \alpha_{\tilde{\sigma}_j{}^*}$ and $\gamma_{\tilde{\sigma}_j} \le \gamma_{\tilde{\sigma}_j{}^*}, \forall j$.

(iv) Commutative: Let $\tilde{\sigma}_j$, (j=1,2,.....,n), be a collection of Linguistic Intuitionistic Fuzzy numbers; Then we have to prove, $\text{LinIFWAA}_\omega(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n) = \text{LinIFWAA}_\omega(\bar{\tilde{\sigma}}_1,\bar{\tilde{\sigma}}_2,...,\bar{\tilde{\sigma}}_n)$, j=1,2,...,n and for all ω where $(\bar{\tilde{\sigma}}_1,\bar{\tilde{\sigma}}_2,...,\bar{\tilde{\sigma}}_n)$ is any permutation of$(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n)$. Let

$$\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n) = \sum_{j=1}^{n} \tilde{\alpha} w_j$$

$$= \left\langle \sum_{j=1}^{n} \theta_j w_j\,, \left[1\text{-}\prod_{j=1}^{n}(1\text{-}\alpha_j)^{w_j}, \prod_{j=1}^{n} \gamma_j{}^{w_j} \right] \right\rangle$$

$$= \left\langle S_{\theta_j}, [\alpha_{\tilde{\sigma}_j}], [\gamma_{\tilde{\sigma}_j}] \right\rangle,$$

Now,

$$\text{LinIFWAA}_\omega\left((\bar{\tilde{\sigma}}_1,\bar{\tilde{\sigma}}_2,...,\bar{\tilde{\sigma}}_n)\right) = \sum_{j=1}^{n} \bar{\tilde{\alpha}} w_j$$

$$= \left\langle \sum_{j=1}^{n} \bar{\theta}_j w_j\,, \left[1\text{-}\prod_{j=1}^{n}(1\text{-}\bar{\alpha}_j)^{w_j}, \prod_{j=1}^{n} \bar{\gamma}_j{}^{w_j} \right] \right\rangle$$

$$= \left\langle S_{\bar{\theta}_j}, [\bar{\alpha}_{\tilde{\sigma}_j}], [\bar{\gamma}_{\tilde{\sigma}_j}] \right\rangle.$$

Since $(\bar{\tilde{\sigma}}_1,\bar{\tilde{\sigma}}_2,...,\bar{\tilde{\sigma}}_n)$ is any permutation of $(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n)$ we can have $\bar{\tilde{\sigma}}_{\sigma(j)} = \tilde{\sigma}_{\sigma(j)}$, forj=1,2,...,n and hence $\text{LinIFWAA}_w(\tilde{\sigma}_1,\tilde{\sigma}_2,...,\tilde{\sigma}_n) = \text{LinIFWAA}_w(\bar{\tilde{\sigma}}_1,\bar{\tilde{\sigma}}_2,...,\bar{\tilde{\sigma}}_n)$.

E. Theorem 3

Let $\tilde{\sigma}_j$, (j=1,2,.....,n), be a collection of Linguistic Intuitionistic Fuzzy numbers and $w = (w_1, w_2, ..., w_n)^T$ be the weight vector of *LinIFWAA* operator, with $w_j \in [0,1]$ and $\sum_{j=1}^{n} w_j = 1$. Then we have the following:

1. If $\omega = (1,0,0,....,0)^T$, then $\text{LinIFWAA}_w(\tilde{\sigma}_1, \tilde{\sigma}_2, ..., \tilde{\sigma}_n) = \max_j(\tilde{\sigma}_j)$,.

2. If $\omega = (0,0,0,....,1)^T$ then $\text{LinIFWAA}_w(\tilde{\sigma}_1, \tilde{\sigma}_2, ..., \tilde{\sigma}_n) = \min_j(\tilde{\sigma}_j)$,

3. If $w_j = 1, w_i = 0$ and $i \neq j$, then $\text{LinIFWAA}_w(\tilde{\sigma}_1, \tilde{\sigma}_2, ..., \tilde{\sigma}_n) = (\tilde{\sigma}_j)$, where $\tilde{\sigma}_j$ is the j^{th} largest of $\tilde{\sigma}_j$, (j=1,2,.....,n).

4|The Delta learning rule in ANN for Intuitionistic Fuzzy Decision-Making

In decision-making problems where uncertainty and vagueness play a significant role, intuitionistic fuzzy sets provide a powerful framework to represent knowledge. Each alternative in such a system is described not only by a degree of membership but also by a degree of non-membership, with the remaining part interpreted as hesitation. This richer representation captures the ambiguity inherent in human judgment more effectively than classical approaches. To process such information and identify the best alternative, artificial neural networks can be employed. The Delta Learning Rule acts as the mechanism through which the neural network adapts its internal weights while learning from the aggregated intuitionistic fuzzy data. The process begins with the preparation of input vectors derived from membership and non-membership matrices. These inputs are combined using aggregation operators to generate a compressed representation of each alternative. The network then applies an activation function to transform these inputs into an output score. This score reflects how well the alternative aligns with the desired target specified by the decision-maker. At this point, the Delta Learning Rule comes into play. The output generated by the network is compared with the expected target, and the difference between the two is treated as the error. The rule then modifies the network's weights in proportion to this error, the responsiveness of the activation function, and the contribution of the input signals. In effect, the rule ensures that alternatives producing large errors are adjusted more strongly, while those already close to the target undergo smaller refinements. Over successive training cycles, the Delta Learning Rule steadily reduces the gap between the actual and desired outputs. This is reflected in the convergence curve, which tracks the decline of error over time. Once the learning stabilises, the network produces reliable outputs for all alternatives. A threshold value, often based on the average weight, can then be applied to distinguish which alternatives meet the required conditions. The final decision is made by selecting the alternative with the highest output score or by identifying those that satisfy the threshold criterion. By integrating the Delta Learning Rule with intuitionistic fuzzy decision data, the neural network gains the ability to handle uncertainty while still converging towards consistent outcomes. This hybrid approach not only strengthens the learning process but also ensures that the evaluation of alternatives remains sensitive to both membership and non-membership information. The result is a flexible and effective decision-support model that balances computational learning with the nuanced structure of intuitionistic fuzzy information.

5|Algorithm for solving Linguistic Intuitionistic Fuzzy MAGDM problems using ANN

Step-1: Input decision data → Membership and non-membership matrices.

Step-2: Aggregate using IF aggregation → produce compressed input vector for ANN.

Step-3: Initialize ANN with weight vector and activation function.

Step-4: Train ANN with Delta Learning Rule → update weights iteratively.

Step-5: Track error → plot MSE vs. epoch (convergence curves).

Step-6: Final outputs → compute ANN scores per alternative.

Step-7: Select best alternative = the one with highest final ANN score (or satisfying threshold).

6|Numerical Illustration

Assume there are four industries (alternatives) $\{L_1,L_2,L_3,L_4\}$ to be weighed against certain criteria. Evaluate industries in terms of their technological innovation capability, evaluating 'factors' such as resource ability for digitalization (C_1) ,organizational innovation (C_2) , Innovation Centers (C_3) and Innovative products (C_4) . The Experts assessment of the four industries are listed in “Tables 1, 2, and 3”. The numerical computations are programmed using Python and the outputs will be presented based on the interpretations arrived using the newly proposed MAGDM-ANN based algorithm above.

TABLE 1 DECISION MATRIX I

ıdustries	Digitalization C_1	Organizational Inovation C_2	Innovation Centers C_3	Innovative products C_4

L_1	$\langle l_5,(0.2,0.7)\rangle$	$\langle l_2,(0.4,0.6)\rangle$	$\langle l_5,(0.5,0.5)\rangle$	$\langle l_3,(0.2,0.6)\rangle$
L_2	$\langle l_4,(0.4,0.6)\rangle$	$\langle l_5,(0.4,0.5)\rangle$	$\langle l_3,(0.1,0.8)\rangle$	$\langle l_4,(0.5,0.5)\rangle$
L_3	$\langle l_3,(0.2,0.7)\rangle$	$\langle l_4,(0.2,0.7)\rangle$	$\langle l_4,(0.3,0.7)\rangle$	$\langle l_5,(0.2,0.7)\rangle$
L_4	$\langle l_6,(0.5,0.4)\rangle$	$\langle l_2,(0.2,0.8)\rangle$	$\langle l_3,(0.2,0.6)\rangle$	$\langle l_3,(0.3,0.6)\rangle$

TABLE 2 DECISION MATRIX II

lustries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_4,(0.1,0.7)\rangle$	$\langle l_3,(0.2,0.7)\rangle$	$\langle l_3,(0.2,0.8)\rangle$	$\langle l_6,(0.4,0.5)\rangle$
L_2	$\langle l_5,(0.4,0.5)\rangle$	$\langle l_3,(0.3,0.6)\rangle$	$\langle l_4,(0.2,0.6)\rangle$	$\langle l_3,(0.2,0.7)\rangle$
L_3	$\langle l_3,(0.2,0.7)\rangle$	$\langle l_4,(0.2,0.7)\rangle$	$\langle l_2,(0.4,0.6)\rangle$	$\langle l_3,(0.3,0.7)\rangle$
L_4	$\langle l_5,(0.3,0.6)\rangle$	$\langle l_4,(0.4,0.5)\rangle$	$\langle l_2,(0.3,0.6)\rangle$	$\langle l_4,(0.2,0.6)\rangle$

TABLE 3 DECISION MATRIX III

Industries	Digitalization C_1	Organizational innovation C_2	Innovation Centers C_3	Innovative products C_4
L_1	$\langle l_5,(0.2,0.6)\rangle$	$\langle l_3,(0.3,0.7)\rangle$	$\langle l_4,(0.4,0.5)\rangle$	$\langle l_4,(0.2,0.7)\rangle$
L_2	$\langle l_4,(0.3,0.7)\rangle$	$\langle l_5,(0.3,0.6)\rangle$	$\langle l_2,(0.1,0.8)\rangle$	$\langle l_3,(0.4,0.6)\rangle$
L_3	$\langle l_4,(0.2,0.7)\rangle$	$\langle l_5,(0.3,0.6)\rangle$	$\langle l_1,(0.1,0.8)\rangle$	$\langle l_4,(0.2,0.7)\rangle$
L_4	$\langle l_3,(0.2,0.7)\rangle$	$\langle l_3,(0.1,0.7)\rangle$	$\langle l_4,(0.3,0.6)\rangle$	$\langle l_5,(0.4,0.5)\rangle$

ANN OUTPUT FOR THE RANKING OF ALTERNATIVES:

```
[array([0.11599]), array([0.07113]), array([0.08222]), array([0.09156])]
[array([0.10177]), array([0.12418]), array([0.04661]), array([0.14853])]
[array([0.09575]), array([0.06302]), array([0.10455]), array([0.14592])]
```

When d1= -1 ;d2= 1 ;d3= -1

Final weight matrix after delta learning rule

```
[[-0.34169526]
 [ 0.15525386]
 [-0.49329736]
 [-0.2351893 ]]
```

The threshold is -0.22873

Alternative A 1 does not satisfy the threshold -0.22873

Alternative A 2 satisfies the threshold -0.22873

Alternative A 3 does not satisfy the threshold -0.22873

Alternative A 4 does not satisfy the threshold -0.22873

When d1= 1 ;d2= -1 ;d3= -1

Final weight matrix after delta learning rule

[[-0.17324788]
 [-0.35011152]
 [-0.11933175]
 [-0.76908204]]

The threshold is -0.35294

Alternative A 1 satisfies the threshold -0.35294

Alternative A 2 satisfies the threshold -0.35294

Alternative A 3 satisfies the threshold -0.35294

Alternative A 4 does not satisfy the threshold -0.35294

When d1= -1 ;d2= -1 ;d3= -1

Final weight matrix after delta learning rule

[[-1.18685938]
 [-0.93927842]
 [-0.8326423]
 [-1.50169902]]

The threshold is -1.11512

Alternative A 1 does not satisfy the threshold -1.11512

Alternative A 2 satisfies the threshold -1.11512

Alternative A 3 satisfies the threshold -1.11512

Alternative A 4 does not satisfy the threshold -1.11512

When d1= 1 ;d2= -1 ;d3= 1

Final weight matrix after delta learning rule

[[0.6333182]

[0.15606858]

[0.81118648]

[0.50043357]]

The threshold is 0.52525

Alternative A 1 satisfies the threshold 0.52525

Alternative A 2 does not satisfy the threshold 0.52525

Alternative A 3 satisfies the threshold 0.52525

Alternative A 4 does not satisfy the threshold 0.52525

Final ANN Outputs (last epoch):

Case a (-1,1,-1): [-0.2351893]

Case b (1,-1,-1): [-0.76908204]

Case c (-1,-1,-1): [-1.50169902]

Case d (1,-1,1): [0.50043357]

Best Alternative Selected: Case d (1,-1,1)

Hence, in case d, the best alternative choices are A1 and A3.

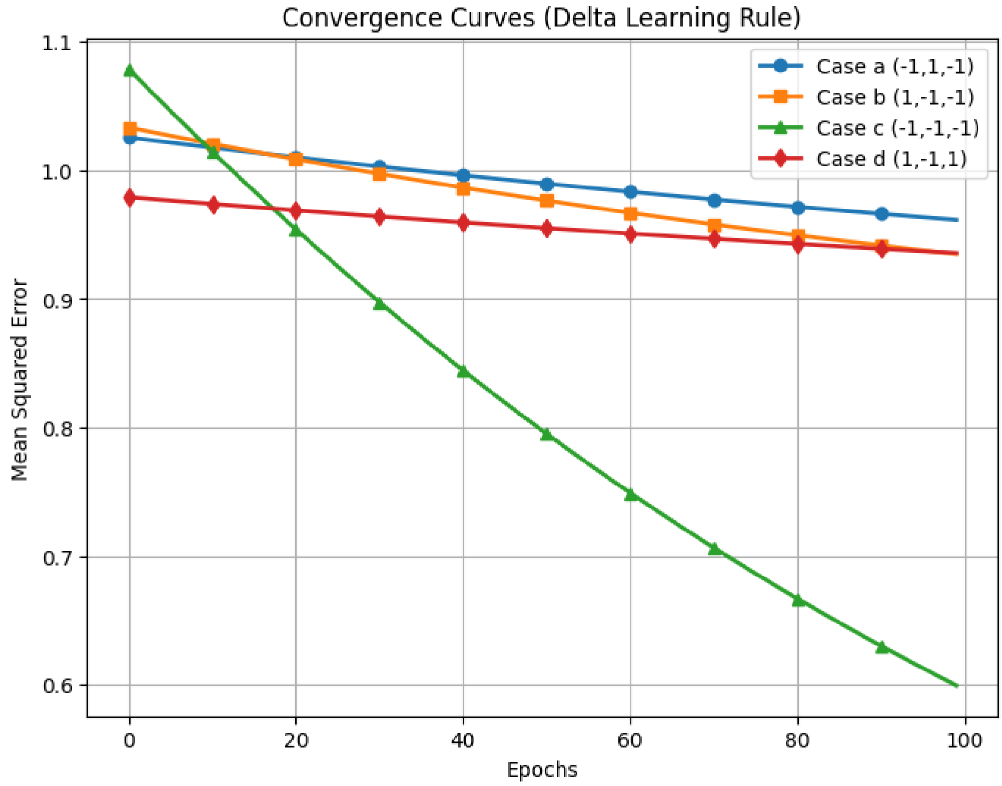

Fig-1: Convergence of ANN Training under Different Target Output Configurations (Delta Learning Rule)

Description of the Convergence Curve:

X-axis (Epochs): Training iterations (from 0 to 100).

Y-axis (Mean Squared Error, MSE): The error between the predicted output and the target.

Curves (Cases a–d): Each corresponds to a different choice of (d1,d2,d3).

Interpretation of each curve:

1. **Case c (-1,-1,-1) [green curve]:**

    This shows the steepest and smoothest decline in error.

    Starts high (~1.08) and decreases steadily below 0.6 by epoch 100.

    Indicates **best convergence**: the network is successfully learning and reducing error consistently.

    This case gives the most reliable training performance.

2. **Case a (-1,1,-1) [blue curve]:**

    Error decreases only slightly (from ~1.03 to ~0.97).

    Plateau after ~30 epochs → learning is very slow, stuck in a shallow minimum.

3. **Case b (1,-1,-1) [orange curve]:**

    Similar to Case a, but a bit better → error decreases from ~1.04 to ~0.945.

    Shows learning, but not strong convergence.

4. **Case d (1,-1,1) [red curve]:**

    Starts lower (0.98) and slowly decreases toward ~0.935.

    Very flat convergence (almost no learning).

Overall Interpretation:

- **Case c (-1,-1,-1) is clearly the best**: strong downward trend, consistent error reduction, no signs of overfitting (since error keeps decreasing without bouncing).
- **Cases a, b, and d** → weak learning, network might be stuck in local minima or weights not well aligned with targets.
- None of the curves show overfitting (no increase in error after an initial decrease).
- But only Case c shows effective training convergence.

Final Conclusion on the convergence: network converges well only in Case c. That means the choice of target outputs (d1,d2,d3) heavily influences learnability and best alternative selection. After training, the ANN produces an output score for each alternative. The best alternative is the one with the highest output score (or lowest error, depending on your formulation). Since we are using the Delta Learning Rule with sigmoid/tanh-type outputs, we can just take the final ANN output values and rank them for the best choice of the alternatives.

Recommendation:

If our decision rule is "pick the alternative with the highest ANN output," we can keep Case d as the winner.

If we want robustness (the winner must also have learned well), use rule B or C above. For example, require final MSE < 0.1 and then pick highest output among those

6|Discussion

The present study employed the Delta Learning Rule in an Artificial Neural Network (ANN) to evaluate multiple alternatives under different target output scenarios. The analysis highlights the critical role of target selection in determining both the learning performance of the network and the resulting ranking of alternatives. The convergence behavior of the ANN varied significantly across the four cases. Case c (-1, -1, -1) demonstrated the most robust learning, characterized by a smooth and steep decline in the Mean Squared Error (MSE) across epochs. This indicates that the network effectively adjusted its weights and minimized prediction errors, reflecting strong learnability for this target configuration. In contrast, Cases a (-1, 1, -1), b (1, -1, -1), and d (1, -1, 1) showed limited error reduction and early plateauing, suggesting weaker convergence and potential entrapment in local minima. Notably, none of the cases exhibited overfitting, as MSE consistently decreased or plateaued without subsequent increases.

Despite the variation in convergence, the network outputs provided a clear basis for alternative selection. Using the final ANN output scores as the decision criterion, Case d (1, -1, 1) emerged as the best scenario, identifying alternatives A1 and A3 as the top choices. This finding underscores an important insight: even when convergence is suboptimal, the ANN can still be useful for ranking alternatives based on relative output values. However, for applications requiring higher reliability and confidence in learned patterns, configurations demonstrating strong convergence, such as Case c, may be preferred. Overall, the results emphasize the dual influence of target outputs on ANN performance: they not only determine which alternatives are selected but also affect the network's ability to learn effectively. These findings suggest that careful consideration of target design is crucial when applying the Delta Learning Rule in decision-making contexts, particularly for multi-criteria evaluation tasks.

7|Conclusion

This study presents an Artificial Neural Network (ANN) approach, using the Delta Learning Rule, to support Multi-Attribute Group Decision-Making (MAGDM) in a commerce and management context. The methodology enables the systematic evaluation of multiple alternatives across several criteria, providing a structured and data-driven framework for informed decision-making. The experiments revealed that the configuration of target outputs significantly influences both the learning performance of the ANN and the reliability of alternative selection. While some cases exhibited strong convergence and stable learning, others demonstrated weaker convergence but still offered useful insights for ranking alternatives. Using the ANN output scores, the most suitable alternatives were successfully identified, illustrating the method's practical applicability for managerial decision-making. Overall, the proposed approach demonstrates that ANN-based MAGDM can effectively support complex business decisions, such as product evaluation, investment selection, or policy prioritization, where multiple attributes and expert judgments must be considered. By integrating computational intelligence with decision-making, organizations can enhance accuracy, consistency, and confidence in their strategic choices.

Author Contributions

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Conflicts of Interest

This is the original work of the authors and all authors have seen and approved the final version of the manuscript being submitted. The material described here is not under publication or consideration for publication elsewhere. "The authors declare no conflict of interest".

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