Some Linear Space Methods for Risk Investment Decisions under Intuitionistic Fuzzy ANN-MAGDM

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Abstract

The aim of this paper is to investigate the Multi Criteria Group Decision Making (MCGDM) or Multiple Attribute Group Decision Making (MAGDM) problems with intuitionistic fuzzy sets. Some Linear space techniques like matrix form of Linear Transformations are used to obtain the solution of fuzzy linear algebraic equations which in turn are utilized to derive the decision maker weights in MAGDM problems under intuitionistic fuzzy sets. In the process of determining weights, multi criteria are explicitly considered, several results based on the above said methods are normalized, and the decision maker's weights for attributes and corresponding decision-making methods have also been proposed. The Artificial Neural Network (ANN) techniques are also utilized for the purpose of decision maker weight determination. Feasibility and effectiveness of the proposed method are illustrated using numerical examples.

Keywords: MAGDM, Linear Transformation, Intuitionistic Fuzzy sets, Weight vector, ANN

Introduction

Artificial Neural Network (ANN) plays a very prominent role in business decision making which is very important in today's common man's life. If one takes wrong decision, sometimes it spoils the whole scenario of the business situation. The decision theory depends on the process of logical and quantitative analysis of all factors that influence the decision problem and assists the decision-maker in analyzing these problems with several courses of action and consequences. To deal with this kind of qualitative, imprecise and incomplete information in decision problems, Zadeh [9] suggested employing the fuzzy set theory as a modeling tool for complex systems. Atanassov [1] introduced the intuitionistic fuzzy sets. Zeng & Li [10] defined the correlation coefficient of intuitionistic fuzzy sets. Group decision making is a participatory process wherein multiple individuals work together to analyze the problem and find out the optimum solution out of the available set of alternatives. Park et al. [4] proposed the correlation coefficient of interval valued intuitionistic fuzzy sets. Decision making is a process of selecting formal choice among the given options. For an effective decision making process, one must be in stable mind to weigh both positive and negative options. Robinson & Amirtharaj [5] described correlation coefficient of triangular and trapezoidal intuitionistic fuzzy sets for multiple attribute group decision making problems. Wei [6] investigated some arithmetic aggregation operators with intuitionistic trapezoidal fuzzy numbers and their application to group decision making. Different families of OWA operators can be used by choosing a different manifestation of the weighting vector (Merigo & Casanovas [3] and Yu [8]). Xu [7] defined intuitionistic preference relations and their application in group decision making. In this work, linear algebraic equations are solved by using bilinear form and linear form and it is clearly described and analysed by Hoffman & Kunze [2]. In this paper, we have investigated the MAGDM problem with intuitionistic fuzzy sets for ranking the alternatives together with the ordered averaging and hybrid averaging operators. The decision maker weights are obtained from different methods like, linear form and bilinear form for determining and normalizing methods for linear algebraic equations through Artificial Neural Network (ANN) techniques are studied. The weights obtained from the methods are applied in decision making problems. A numerical illustration is given to show the effectiveness of the proposed approach.

Preliminaries

In this section, some basic definitions and arithmetic aggregation operators of Intuitionistic Fuzzy Numbers are presented.

Definition 1: Intuitionistic Fuzzy Set

Let a set X be fixed. An IFS Ã in X is an object having the form: $\tilde{A} = \{(x, \mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x)), x \in X\}$, where $\mu_{\tilde{A}}(x): X \to [0,1]$ and $\gamma_{\tilde{A}}(x): X \to [0,1]$. Where $\mu_{\tilde{A}}(x), \gamma_{\tilde{A}}(x)$ define the degree of membership and degree of non-membership respectively, of the element x∈X to the set Ã, which is a subset of X, for every element x ∈ X, $0 \leq \mu_{\tilde{A}}(x) + \gamma_{\tilde{A}}(x) \leq 1$.

Correlation Coefficient of Intuitionistic Fuzzy Sets (IFSS)

In this paper, the method proposed by Zeng& Li, (2007) for calculating correlation coefficient of IFSs is presented, taking only the membership and non-membership grades into account. For $A=\{(\mu_A(x),\ \gamma_A(x),\ \pi_A(x))/x\in X\}$, $B=\{(\mu_B(x),\ \gamma_B(x),\ \pi_B(x))/x\in X\}$, the correlation of the IFSs is defined as:

$$C_{ZL}(A,B)=\frac{1}{n}\sum_{i=1}^{n}\left[\mu_A(x_i)\mu_B(x_i)+\gamma_A(x_i)\gamma_B(x_i)+\pi_A(x_i)\pi_B(x_i)\right]$$

Furthermore, the correlation coefficient of the IFSs, A and$B$ is given by:

$$\rho_{ZL}(A,\ B)=\frac{C_{ZL}(A,\ B)}{\sqrt{C_{ZL}(A,A)\ .\ C_{ZL}(B,B)}}\ .$$

The following proposition and theorems are true for the above defined correlation coefficient.

Proposition 1: For A, $B\in$ IFS(X), we have:

i) $0\le C_{ZL}(A,B)\le 1,$

ii) $C_{ZL}(A,B)=C_{ZL}(B,A),$

iii) $\rho_{ZL}(A,B)=\rho_{ZL}(B,A),$

Theorem 1: For A, $B\in$ IFS(X), then $0\le\rho_{ZL}(A,\ B)\le 1.$

Theorem 2: $\rho_{ZL}(A,\ B)=1\Leftrightarrow A=B.$

Theorem 3: $C_{ZL}(A,\ B)=0\Leftrightarrow A$ and B are non-fuzzy sets and satisfy the condition $\mu_A(x_i)+\mu_B(x_i)=1$ or $\gamma_A(x_i)+\gamma_B(x_i)=1$ or $\pi_A(x_i)+\pi_B(x_i)=1,\ \ \forall\ x_i\in X.$

Theorem 4: $C_{ZL}(A,\ A)=1\Leftrightarrow A$ is a non-fuzzy set.

An Approach to Group Decision Making with Intuitionistic Fuzzy Information

Step 1: Utilize the decision information given in the intuitionistic fuzzy decision matrix $\tilde{R}_k$, and the IFWAA operator,

$$\tilde{r}_i^{(k)}=\left(u_i^{(k)},v_i^{(k)}\right)=IFWA=\left(\tilde{r}_{i_1}^{(k)},\tilde{r}_{i_2}^{(k)},\ldots,\tilde{r}_{i_n}^{(k)}\right),i=1,2,\ldots m;\ \ k=1,2,\ldots t.$$

To derive the individual overall preference intuitionistic fuzzy values $\tilde{r}_i^{\ k}$ of the alternative A_i.

Step 2: Utilize the IFHA operator, $\tilde{r}_i=(\mu_i,\gamma_i)=IFHA_{v,w}=\left(\tilde{r}_i^{(1)},\tilde{r}_i^{(2)},\ldots,\tilde{r}_i^{(t)}\right),i=1,2,\ldots,m.$

To derive the collective overall preference intuitionistic fuzzy values $\tilde{r}_i\left(i=1,2,\ldots..m\right)$ of the alternative Aᵢ where $v=\left(v_1,v_2\ldots v_n\right)$ be the weighting vector of decision makers, with: $V_k\in[0,1],\quad \sum_{k=1}^{t}V_k=1;\;\; w=\left(w_1,w_2\ldots w_n\right)$ is the associated weighting vector of the IFHA operator with $w_j\in[0,1],\quad \sum_{j=-1}^{n}w_j=1.$

Step 3: Calculate the correlation coefficient between the collective overall preference values r_i and the positive ideal value $\tilde{r}_i$, where $\tilde{r}_i=(0,1)$.

The correlation of $A,B\in IFSs(x)$ is given by a formula

$$C_{ZL}\left(A,B\right)=\frac{1}{n}\sum_{i=1}^{n}\left[u_A\left(x_i\right)u_B\left(x_i\right)+\gamma_A\left(x_i\right)\gamma_B\left(x_i\right)+\pi_A\left(x_i\right)\pi_B\left(x_i\right)\right].$$

Step 4: Calculate the correlation coefficient between the collective overall preference values r_i and the positive ideal value $\tilde{r}_i$, where $\tilde{r}_i=(0,1)$.

The correlation coefficient of the IFSs, $A,B\in IFSs(x)$ is given by the formula:

$$\rho_{ZL}\left(A,B\right)=\frac{C_{ZL}\left(A,B\right)}{\sqrt{C_{ZL}(A,A)C_{ZL}(B,B)}}.$$

Step 5: Rank all the alternatives $A_i\left(i\ =1,2,\ldots\ ,m\right)$ and select the best one in accordance with the correlation coefficient obtained in step 4.

Representation of Linear Transformations by Matrices

Let $\{e_1,e_2,\ldots.e_n\}$ be a basis for a vector space V over a field K. For any $v\in V$, there exists unique scalars $\alpha_i\in k$ such that $v=\sum_{i=1}^{n}a_ie_i$.

Write $[v]_e=\begin{bmatrix}a_1\\a_2\\\cdots\\a_n\end{bmatrix}$. Then the scalars a_i are called co-ordinates of v relative to the basis $\{e_i\}$. $[v]_e$ is defined as co-ordinate vector of v relative to the basis $\{e_i\}$

Definition:2 Similar matrices

Let A and B be square matrices of order n over the same field F. Then B is said to be similar to A if there exists an $n\times n$ invertible matrix P over the field F such that

$B = P^{-1}AP$.

Definition:3 Determinant of a linear transformation

Let T be a linear map on n-dimensional vector space V(F).Also let s and s' be two ordered bases of V; then $[T]_s$ = matrix representation of T relative to s.

And $[T]_{s'}$ = matrix representation of T relative to s', are similar matrices. But similar matrices have the same determinant. Hence determinant of matrix $[T]_s$ = determinant of matrix $[T]_{s'}$

Definition 4: Suppose T is a linear transformation on a finite dimensional vector space V(F), then the determinant of T is the determinant of matrix of T relative to any ordered basis of V is, $\det(T) = \det[T]$.

Definition:5 Similarity of linear transformations

Let T_1 and T_2 be linear transformations on the same vector space V (F).Then T_1 is said to be similar to T_2. If there exists an invertible linear transformation T on V (F) such that $T_1 = TT_2T^{-1}$. The relation of similarity of linear transformation in the set v of linear transformation is an equivalence relation.

Theorem 5: Let T be a linear map from an n-dimensional vector space U into an m-dimensional vector space V and let B and B' be ordered bases for U and V respectively .If A is a matrix of T relative to B and B' then

$\forall \alpha \in U$ we have $[T(\alpha)]_{B'} = A[\alpha]_B, i, e., [T(\alpha)]_{B'} = [T]_B^{B'}[\alpha]_B$.

Where $[\alpha]_B$ is the co-ordinate vector of α with respect to ordered basis B and $[T(\alpha)]_{B'}$ is the co-ordinate vector of $T(\alpha) \in V$ $w.r.tB'$

Decision Maker Weight Information in the Form of Linear Transformation

Let $V = R^3$and $T: V \to V$ be a linear map defined $T(x, y, z) = (x + y, -2x + y, -x + 2y + z)$ let us determine matrix of T with respect to basis $\{(1,0,1), (-1,1,1), (0,1,1)\}$.

$T(x_1, x_2, x_3) = (3x_1 + x_3, -2x_1 + x_2, -x_1 + 2x_2 + 4x_3)$ and $\{e_1, e_2, e_3\}$ is a basis of R^3, where, $e_1 = (1,0,1)$; $e_2 = (-1,1,1)$; $e_3 = (0,1,1)$. Then,

$$T(e_1) = T(1,0,1) = (2,-2,0)$$
$$T(e_2) = T(-1,1,1) = (0,3,4)$$
$$T(e_3) = T(0,1,1) = (1,1,3)$$

Let $(x, y, z) = ae_1 + be_2 + ce_3$

Then, $(x, y, z) = [(a-b), (b+c), (a+b+c)]$

$$a - b = x, b + c = y, a + b + c = z; a = z - y, b = z - y - x, c = x + 2y - z$$

Then for specific values of x, y and z we have:

$$T(e_1) = 2e_1 + 0e_2 - 2e_3$$
$$T(e_2) = e_1 + e_2 + 2e_3$$
$$T(e_3) = 2e_1 + e_2 + 0e_3$$

Then, $w = \begin{pmatrix} 2 & 0 & -2 \\ 1 & 1 & 2 \\ 2 & 1 & 0 \end{pmatrix}$; and we know that $A = \begin{pmatrix} 1 & 0 & 1 \\ -1 & 1 & 1 \\ 0 & 1 & 1 \end{pmatrix}$.

Then, $(A^T w)^T (A^T w) = \begin{pmatrix} 35 & 15 & 2 \\ 15 & 9 & 8 \\ 2 & 8 & 20 \end{pmatrix}$.

Taking column normalization for the above matrix, we get:

$$w = \begin{pmatrix} 0.673076923 & 0.4687500 & 0.06666 \\ 0.288461538 & 0.2812500 & 0.266667 \\ 0.038461538 & 0.25000 & 0.66667 \end{pmatrix}$$

Application of Artificial Neural Network for Deriving the Decision Maker Weight Vector

A simple architecture of the Artificial Neural Network (ANN) is presented in the following diagram where two inputs are processed and the aggregated value is passed on to the activation function to get the desired output which is nearest to the target value.

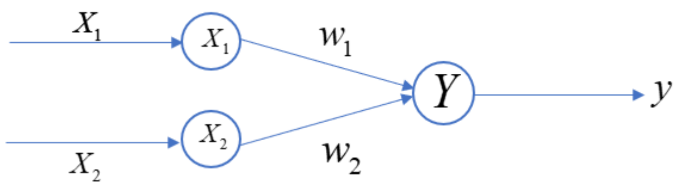

Fig-1: Simple Architecture of a Neural Net

Considering the following assumptions to proceed with the ANN process, we need the:

i) Input Vector – the 3x3 matrix W,

ii) Weighted Matrix computed from the matrix W,

iii) Activation function – Sigmoid function,

iv) No biases for the model to derive the decision maker weighting vector, and

v) Initial random weight vector assigned by the Neural Net for the computations.

Hence, performing a simple Neural Network for the matrix W, we get the following computations:

The weighted sum is

$$Z = \begin{pmatrix} 0.5908 & 0.4640 & 0.2143 \\ 0.2855 & 0.2810 & 0.2720 \\ 0.1236 & 0.2550 & 0.5137 \end{pmatrix}.$$

Now applying the Sigmoid function we obtain the following matrix:

$$Y = \begin{pmatrix} 0.6436 & 0.6140 & 0.5534 \\ 0.5709 & 0.5698 & 0.5676 \\ 0.5309 & 0.5634 & 0.6257 \end{pmatrix}$$

Now, allowing the Neural Net to produce one scalar output for each row vector, we get:

Output for Row 1: 0.7806, Output for Row 2: 0.6982, Output for Row 3: 0.7094. Then, normalizing the above output, we get: $\gamma = (0.3567,\ 0.3191,\ 0.3242)$ as the weight vector of the decision maker, which will be used in further computations of solving the MAGDM problem.

MAGDM with Application of Linear Transformation

Let us suppose there is a risk investment company, which wants to invest a sum of money in the best option. There is a panel with five possible alternatives to invest the money. The risk investment company must take a decision according to the following four attributes:

G1 is the risk analysis.

G2 is the growth analysis.

G3 is the social-political impact analysis.

G4 is the environmental impact analysis.

The five possible alternatives $A_i(i = 1,2,3,4,5)$ are to be evaluated using intuitionistic fuzzy numbers by the three decision makers whose weighting vector is obtained

by normalizing the solution of linear transformation process through Artificial Neural Network (ANN) without any bias is $\gamma = (0.3567,\ 0.3191,\ 0.3242)^{T}$, nder the above four attributes whose weighting vector$\omega = (0.2, 0.1, 0.3, 0.4)^T$and construct, respectively, the decision matrices as listed in the following matrices $R = \left(r_{2ij}^{(k)}\right) 5 * 4 (k = 1,2,3)$As follows:

$$R_1 = \begin{pmatrix} (0.5, 0.4) & (0.6, 0.3) & (0.3, 0.6) & (0.2, 0.7) \\ (0.7, 0.3) & (0.7, 0.2) & (0.7, 0.2) & (0.4, 0.5) \\ (0.6, 0.4) & (0.5, 0.4) & (0.5, 0.3) & (0.2, 0.3) \\ (0.8, 0.1) & (0.6, 0.3) & (0.3, 0.4) & (0.2, 0.6) \\ (0.6, 0.2) & (0.4, 0.3) & (0.7, 0.1) & (0.1, 0.3) \end{pmatrix}$$

$$R_2 = \begin{pmatrix} (0.4,0.3) & (0.5,0.2) & (0.2,0.5) & (0.1,0.6) \\ (0.6,0.2) & (0.6,0.1) & (0.6,0.1) & (0.3,0.4) \\ (0.5,0.3) & (0.4,0.3) & (0.4,0.2) & (0.5,0.2) \\ (0.7,0.1) & (0.5,0.2) & (0.2,0.3) & (0.1,0.5) \\ (0.5,0.1) & (0.3,0.2) & (0.6,0.2) & (0.4,0.2) \end{pmatrix}$$

$$R_3 = \begin{pmatrix} (0.4,0.5) & (0.5,0.4) & (0.2,0.7) & (0.1,0.8) \\ (0.6,0.4) & (0.6,0.3) & (0.6,0.3) & (0.3,0.6) \\ (0.5,0.5) & (0.4,0.5) & (0.4,0.4) & (0.5,0.4) \\ (0.7,0.2) & (0.5,0.4) & (0.2,0.5) & (0.1,0.7) \\ (0.5,0.3) & (0.3,0.4) & (0.6,0.2) & (0.4,0.4) \end{pmatrix}$$

Step1: Utilize the decision information given in the intuitionistic fuzzy decision matrix $\tilde{R}_k$, and the IFOWA operator to derive the individual overall preference intuitionistic fuzzy values $\tilde{r}_i(k)$ of the alternative A_i. Hence, we get:

$r_1^{(1)} = (0.347222154, 0.549045978)$ $r_2^{(1)} = (0.604147627, 0.312913464)$

$r_3^{(1)} = (0.422920038, 0.327041507)$ $r_4^{(1)} = (0.456527711, 0.346410162)$

$r_5^{(1)} = (0.471481123, 0.198960391)$ $r_1^{(2)} = (0.244644753, 0.443076536)$

$r_2^{(2)} = (0.499648495, 0.2)$ $r_3^{(2)} = (0.462173122, 0.225869387)$

$r_4^{(2)} = (0.342425064, 0.283679124)$ $r_5^{(2)} = (0.479785681, 0.174110113)$

$r_1^{(3)} = (0.244644753, 0.652777846)$ $r_2^{(3)} = (0.499648495, 0.419296271)$

$r_3^{(3)} = (0.462173122, 0.427693840)$ $r_4^{(3)} = (0.342425064, 0.465738581)$

$r_5^{(3)} = (0.479785681, 0.306734937)$

Step 2: Utilize the IFHA operator to derive the collective overall preference intuitionistic fuzzy values $\tilde{r}_1$ of the alternative A_i. Then we get,

$r_1 = (0.281077587, 0.558035935)$, $r_2 = (0.537751612, 0.312198869)$,

$r_3 = (0.447818157, 0.332945257)$, $r_4 = (0.383013746, 0.373806100)$,

$r_5 = (0.493901507, 0.232976226)$.

Step 3: Calculate the correlation coefficient between the collective overall preference values r_i and the positive ideal value $\tilde{r}_i$, where $\tilde{r}_i = (0,1)$.

Hence the calculated values are given as follows:

$C_{ZL}(r_1, \tilde{r}_1) = 0.558035835$, $C_{ZL}(r_2, \tilde{r}_2) = 0.312198869$

$C_{ZL}(r_3, \tilde{r}_3) = 0.332945257$, $C_{ZL}(r_4, \tilde{r}_4) = 0.373806100$

$C_{ZL}(r_5, \tilde{r}_5) = 0.232976226$.

Step 4: By using the algorithm in step 4, we get:

$\rho_{ZL}(r_1, \tilde{r}_1) = 0.864893279, \rho_{ZL}(r_2, \tilde{r}_2) = 0.488073078$

$\rho_{ZL}(r_3, \tilde{r}_3) = 0.555326704, \rho_{ZL}(r_4, \tilde{r}_4) = 0.635911906$

$\rho_{ZL}(r_5, \tilde{r}_5) = 0.381563159.$

Step 5: Rank all the alternatives $A_i(i = 1,2,3,4,5)$from the highest closeness (correlation coefficient) obtained from step 4, the result is as follows: $A_1 > A_4 > A_3 > A_2 > A_5$

Hence, the best alternative is A_1.

Conclusion

In this work, we have discussed about the weight determining methods together with weighted averaging operator and the ordered weighted averaging operator, which extend two of the most common aggregation operators to accommodate the situations where the input arguments are interval valued intuitionistic fuzzy values. New methods based on some Linear space techniques such as bilinear forms and quadratic forms are used for determining the unknown decision maker weights. The proposed approach in this work not only can comfort the influence of unjust arguments on the decision results, but also avoid losing or distorting the original decision information in the process of aggregation. Thus, the proposed approaches provide us an effective and practical way to deal with multi-person multi-attribute decision making problems, where the attribute values are characterized by intuitionistic fuzzy numbers and the information about decision maker weights is completely unknown.

Author Contributions

"Conceptualization, Sumithra G and John Robinson P; Methodology, Sumithra G; Software, John Robinson P; Validation, Sumithra G and John Robinson P; formal analysis, Sumithra G; investigation, John Robinson P; resources, Sumithra G; data maintenance, John Robinson P; writing-creating the initial design, Sumithra G; writing-reviewing and editing, Sumithra G; visualization, John Robinson P; monitoring, John Robinson P; project management, John Robinson P. All authors have read and agreed to the published version of the manuscript. Authorship must be limited to those who have made a significant contribution to the work reported.

Conflicts of Interest

This is the original work of the authors and all authors have seen and approved the final version of the manuscript being submitted. The material described here is not under publication or consideration for publication elsewhere. “The authors declare no conflict of interest”.

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